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Scoring Rules*

F. Dini, R. Pacini, and T. Valletti[♣]

Abstract

Competition for procurement contracts has a multidimensional nature. Suppliers' tenders comprise both an economic and a qualitative/technical aspect. The winner of a competitive tendering is determined by assigning a score to each dimension of an offer. The score can be calculated using a scoring rule, which is a mathematical formula that allows the buyer to rank offers and determine the winning supplier. In this paper we investigate several practical issues concerning scoring rules that should inform the choices of procurers. We first discuss how the procurer should choose the scoring rule according to her preferences. This illustrates how to solve the possible tension between low prices and high quality. We also identify the settings where a particular scoring rule is likely to work better than others do.

Keywords: procurement, auctions, scoring rules.
Jel classification: H57, D44.

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1. INTRODUCTION

Competition for procurement contracts is widely recognized to have a multidimensional nature. The buyer often cares about both the price and other non-monetary attributes that include various measures of quality. A common format that can be used in these circumstances is a "scoring auction". In a scoring auction participants bid for price and non-price attributes (quality). The buyer then selects the winner using a scoring rule that weights price and quality, with the aim of achieving best-value-for-money. Hence, scoring auctions involve the choice of a scoring rule that allows the buyer to rank offers and to determine the final contractor. Scoring auctions can be more costly to design and run compared to price-only auctions, since they require the evaluation of potentially complex quality attributes. However, they also guarantee more flexibility when handling the trade-off between price and quality.

In this paper we investigate several practical issues concerning scoring rules that should inform the choices of procurement bodies. First, we ask how the procurer should choose the scoring rule according to her preferences. This illustrates how to solve the possible tension between low prices and high quality. Second, we describe different classes of scoring rules and we analyse their properties paying particular attention to their simplicity, predictability, and ability to promote competition. We identify the settings where a particular scoring rule is likely to work better than others do, providing some relevant Practical Conclusions for procurement bodies. Third, we discuss the problem of abnormally low tenders, and show how scoring rules can be adapted to prevent problems associated to extremely low bids.

2. WHAT ARE SCORING AUCTIONS?

Procurement contracts for goods and services are usually auctioned taking into account some non-price attributes that include various measures of quality. We refer to these attributes simply as "technical aspects" to emphasize the fact that they are quite easy to measure and contract upon. For instance, in the procurement of personal computers, the hard-disk, the screen dimension, the weight, etc., are all valuable technical aspects for end-users, thus the buyer is likely to take them into account in the auction design. The assessment of relevant technical aspects in a procurement contract involves a weighting scheme allowing the buyer to award the contract not simply to the lowest price but to the best price-quality combination.[1] Weighting technical aspects implies the set-up of a *scoring rule*, that is, a mechanism assigning a score to each dimension of the contract. The sum of these scores determines the bidder's total score. The total score is the crucial "number" upon which suppliers' price-quality offers are compared in order to determine the winner (i.e., the contractor).

Practical Conclusion 1

When quality is a crucial component of a procurement contract and you look for flexibility to handle the trade-off between price and quality, scoring auctions are particularly appropriate

[1] The European directive refers to this as the "most economically advantageous" offer.

By way of illustration, suppose a buyer wants to procure screens for PCs, taking into account only the price and the dimension of the screen. This is a two-dimensional procurement auction for which the scoring rule will be the following:

Total score = price score + screen dimension score

The total score is composed of two "sub-scores", one for each dimension of the contract. Sub-scores are essentially weights that the buyer attaches to the attributes evaluated.

The scoring rule should reflect the buyer preferences, namely the relative importance of each aspect considered in the contract. In the spirit of the example illustrated above, if the dimension of the screen (quality) is perceived as very important, the buyer will attach a considerable weight to it (e.g., she will attribute many technical points to large screens). Instead, if price matters much more, for instance because the main buyer's concern is cost saving, the buyer will attach a large weight to price and reduce the importance of quality. In general, technical aspects are typically relatively more important in the procurement of services, non-standardized goods or products with relevant complementary services (assistance, maintenance, etc.).

2.1. PRICE-QUALITY TRADE-OFF IN PROCUREMENT AUCTIONS

Imagine one bid is first on *all* dimensions of the contract, i.e., a bidder offers both the cheapest price and the best technical performance. This bidder will win the contract under different scoring rules, even if they differ in their weightings. However, this situation is not a general one since *price-quality trade-offs* are likely to exist. Indeed, high-quality goods and services usually involve higher costs, so that good technical offers are more likely to be associated to high price offers. Low-quality goods and services involve the opposite situation and are likely to be offered at a cheap price. Since buyers want to procure valuable products at the lowest price, quality and price are conflicting goals. Therefore, buyers face a price-quality trade-off between cheap-low-quality and expensive-high-quality products and it is expected that different scoring rules determine different rankings of the same price-quality combinations and then different winners. This is shown in the following example.

> **Example 1.** Three bidders, A, B and C participate in a procurement auction with a reserve price of €100. The price offers are expressed in percentage discount on the reserve price. There is a total of 100 points available, 50 for the price offer and 50 for the technical aspects. The price score is given by the rule: "(% discount)*50". Suppose the bidders offer the following discounts: 10% (A), 15% (B) and 35% (C) and that the technical points obtained are 20 (A), 10 (B) and 7 (C). These offers yield the following final ranking: A is first (25 total points), C comes second (24.5 points) and B comes third (17.5 points). Now, suppose the price scoring rule is changed to: "(Lowest bid/Price bid)*50" and that the bidders still offer the same bids as before. It is easy to verify that with the new scoring rule C is first (57 total points), A comes second (56.1 points) and B is third (48.2 points). Therefore, different scoring rules can determine different final rankings.

To solve the price-quality trade-off, the buyer should look at her preferences and set the scoring rule accordingly. In the next sections we provide some indications useful to manage this problem. For this purpose we address the following questions: what amount of money is the buyer willing to pay for quality? What price-quality combinations can be considered equivalent from the standpoint of the buyer? What price discount does the buyer require to award an extra point? The process through which the scoring rule is chosen and calibrated is just the way the buyer answers these questions.

2.2. THE MONETARY VALUE OF A POINT (MVP) AND THE BUYER'S MONETARY EQUIVALENT (BME)

What amount of money is the buyer willing to pay for quality? What price discount does the buyer require to award an extra point? These are the key questions the buyer faces when designing a multidimensional auction. We show how the respective answers to both questions, namely the *buyer's monetary equivalent* for quality (henceforth, BME) and the *monetary value of a point* (henceforth, MVP), allow to solve the price-quality trade-off. We start from the concept of MVP, which is easy to grasp by way of a simple example.

> **Example 2.** A buyer has to choose a contractor for the provision of a good/service and has to evaluate economic offers (price offers) and quality (technical offers). She sets a total of 100 points available, 50 for the price offer and 50 for the technical aspects. Suppose bidder A gets 50 points for a price of €100,000, while bidder B gets 40 points for a price of €110,000 (we do not consider yet the technical aspects). Bidder A is awarded 10 extra points for a discount of €10,000 on B's bid: this is as if each point is worth €1,000. Thus, €1,000 is the monetary value of a point in this example.

The MVP is the money discount necessary for a bidder to obtain one additional point. Therefore, the notion of MVP has several important implications. First, the buyer reveals the points value of a discount. Second, the MVP incorporates the buyer's preferences since it allows calculating the monetary value the buyer attaches to non-price attributes. In the example above the MVP (€1,000) would be multiplied by all the points allocated to non-price attributes (a maximum of 50 points), yielding €50,000. This value is the *buyer's monetary equivalent* (BME) of all the technical aspects taken into consideration in the auction. As we shall see, the knowledge of BME is the first necessary step for the buyer to set the scoring rule since it allows to rank price-quality combinations, and to define the MVP in the scoring rule.

Practical Conclusion 2

When you are able to *a priori* evaluate the non-price attributes considered in the auction, you can set the scoring rule accordingly so that a point assigned to bidders on the price-side is monetary equivalent to a point assigned on technical aspects.

The procedure to follow in order to apply Practical Conclusion 2 is simple:

1. set the total points available and divide them between the price and non-price aspects according to the relative importance you give them;
2. evaluate your BME of each non-price attribute;
3. allocate the points you set for non-price aspects in step 1 to each non-price attribute proportionally to its own BME;
4. divide the BME of all non-price attributes by the points assigned to non-price aspects to obtain the MVP;
5. set the price scoring rule so that every point costs the bidders the above MVP (consistency).

This procedure makes it possible for all technical aspects to have the same MVP. The price score is such that the monetary discount necessary to obtain one additional point from price reductions equals the money necessary to obtain (at the buyer's valuation) one additional point from technical improvements.

It is always possible to determine the MVP in any procurement auction once the buyer has received and assigned a score to all the bids (i.e., *ex-post*, as we did in Example 2). However, not all the scoring rules allow to fix the MVP, as recommended above in step 5, so that it is possible for bidders to know it *a priori*. Indeed, the possibility for *a priori* knowledge of the MVP depends on the scoring rule.

Practical Conclusion 3

Favour scoring rules which allow to calculate the MVP *a priori* to promote predictability

When a scoring rule allows to *a priori* calculate the MVP[2], bidders are provided with a *clear* and *certain* relationship between the price bid and the score (predictability): if the score associated to any price offer is known before the auction takes place, participants can easily calculate the MVP and bid accordingly. By comparing his monetary cost of a technical point with the MVP, a bidder can optimally allocate his budget for the auction among all attributes that are subject to assessment. Consider the following example.

Example 3. Consider a multidimensional auction to procure a screen for PCs where the non-price dimension concerns the inches of the monitor. Suppose the buyer adopts a scoring rule which attributes up to 30 points to the non-price attribute and up to 70 points to the price offered. The former are awarded to the technical improvements in this way: 0 points to a 14" monitor, 10 points to a 15" monitor, 20 points to a 16" monitor, 30 points to a 17" monitor. The latter are awarded to price discounts in this way: one point is given for every reduction of €50 in the price bid. This setting then assumes that the buyer's monetary equivalent (BME) for one additional inch is €500. Table 1 illustrates the above weighting scheme.

Table 1 here

We assume the participants define their bidding strategy by allocating their budget for the auction among price and non-price attributes. Consider a representative supplier firm, A. Suppose that A's internal cost reporting says that every inch added starting from 14" costs €400, and that the Head Office of A gives to the sales manager a budget of €1,000 to spend to win the auction.[3] The objective of the manager is to allocate the budget he decides to spend in the auction in the way that achieves the highest net score for A. Suppose he strategically decides to spend the whole budget. Then he finds that the best offer for A consists in a €200 price discount plus a 16" monitor, since the internal cost for an extra technical point is €40, which is less than €50 (the cost for obtaining an extra point from the price side). If instead of €400, every inch added had cost more than €500, the sales manager would have found the best offer for A to consist in a €1,000 price discount, thus spending the whole budget on price discounts rather than on technical improvements.

[2] This is the case of all those scoring rules where a bidder's score does not depend on the bids submitted by other bidders.

[3] Here we abstract from arguments of optimal strategic behaviour relating to auction formats.

Predictability benefits the buyer too, since it reduces the risk of losing potential best value for money, as shown in the following example.

> **Example 4.** Consider the same setting of Example 3, where the buyer is indifferent between two offers that differ in price by €500 and for 1" monitor: that is, an offer of A consisting in a 14" monitor at €2,000 and an offer of B consisting in a 15" monitor at €2,500 are equivalent for the buyer. Therefore, the buyer should keep the scoring rule with the above setting if and only if her actual monetary equivalent (BME) for one additional inch is €500. Otherwise, this scoring rule would not reflect the buyer's preferences, thus raising the risk of losing potential best value for money. In fact, if her 'true' BME for one additional inch was €300 instead, and in any case the winning bidder was the representative supplier firm A of Example 3 with a budget of €1,000 and internal cost of €400 per inch, the buyer would have lost €200. This is because, if the scoring rule implies a MVP of €50, thus implicitly yielding a BME of €500, A offers €200 in price discounts plus a 16" monitor, which has a value for the buyer of €800 (€200 + €300 + €300). If the scoring had assumed instead the correct BME of €300, the A would have offered a €1,000 price discount for a 14" monitor (hence, potential lost savings are €1,000 - €800 = €200).

Finally in the following example we show why scoring rules which do not allow to *a priori* calculate the MVP[4] may harm both the bidders and the buyer.

> **Example 5.** Consider the same setting of Example 3, that is up to 30 points to the non-price attribute and up to 70 points to the price, and the same buyer indifferent between two offers that differ in price by €500 and for 1" monitor (that is, she values each inch added €500), except for the scoring scheme concerning the price offers. Assume there are two bidders, A and B, which have respectively a budget of €1050 and €900 to spend in the auction, and production costs equal to €550 and €350 when they add an inch to a monitor (thus one technical point costs €55 to A and €35 to B). Suppose the price score now depends on the "state of the world" which occurs ex post, for instance because the scoring rule adopted depends on the price bid by both firms. This implies the MVP is unknown *a priori* as it depends on the state of the world realised ex post. Thus, suppliers' bidding strategies can only rely upon "expectations" about the state of the world. Suppose, for instance, there are only two possible states of the world, say high ("H") and low ("L"), to which it corresponds a MVP of €50 and €25, respectively. Uncertainty on the MVP may harm both bidders and the buyer. Suppose after bids are open the state of the world which occurs is that one consistent with the BME, i.e., H. If both suppliers had the right expectations on the average of prices (i.e., "H"), A would offer €1050 in price discounts, B would offer €200 in price discounts plus a 16" monitor, and the buyer would choose B since €1050 < €1200 (= €500 + €500 + €200). But if both suppliers expected the average of prices to be "L", A would still offer €1050 in price discounts, B would instead offer €900 in price discounts, and the buyer would choose A since €1050 > €900. Hence, uncertainty on the MVP can make the buyer loose potential value for money if bidders expect "L" instead of "H". Furthermore, if the state of the world which occurs is L, thus not consistent with the BME, then we fall in what described in Example 4.

2.3. EX-POST EVALUATION OF NON-PRICE ATTRIBUTES

There are circumstances when the buyer can also opt for ex-post evaluation of some or all non-price attributes. Ex post evaluation is involved whenever automatic scoring is not possible, i.e., the

[4] For example, this occurs when the scoring rule depends on the minimum (or the maximum, the average, etc.) bid submitted.

precise way points are awarded to technical improvements and non-price attributes is not defined completely before the auction takes place. This method can be relevant when the buyer is not sufficiently informed on some non-price attributes to give them the right relative weight when designing the scoring rule. In this case it could be better to wait for bidders' offers, from which to gather ex-post more information about the underlying market, and avoid to use an automatic scoring, in order to maintain ex-post some degree of flexibility. Besides this buyer's ex-post flexibility, bidders as well can derive some advantages when the non-price attributes are only indicated in broader terms. Indeed, they have more freedom to enrich and articulate their offers, especially for the procurement of particularly complex goods/services. The downside is that this creates room for discretion.

Detailed *ex-ante* specifications increase the predictability of procurement competition, minimizing risks of discretionary behaviour and abuses of the evaluating committee (e.g., corruption). This comes at the loss of some flexibility. Therefore, the buyer should carefully evaluate and compare case by case costs and benefits of both flexibility and predictability. When corruption is not a particular concern and participants can offer different valuable solutions for the good or service to be procured, it is more likely that benefits from flexibility prevail. In this case, there can be much to gain from ex-post evaluation. In contrast, if corruption during the evaluation process is a concern, ex-post flexibility should be minimized either by adopting an ex-ante BME (automatic scoring) or, in case this were not possible, by reducing the weight given to technical attributes.

Practical Conclusion 4

When opting for ex-post evaluation of non price attributes, consider the trade-off between flexibility and predictability. If corruption during the evaluation process is of potential concern, it is more appropriate to reduce either ex-post evaluation flexibility through automatic scoring or the weight attributed to technical aspects.

Ex-post evaluation of non-price attributes does not make it possible to follow the mechanical procedure we proposed to apply Practical Conclusion 2. Under this alternative scenario, committees in charge of evaluating proposals should operate in a modified way, still assigning points to bidders on the price-side that are monetary equivalent to points assigned on technical aspects. Technical aspects should be evaluated first, before opening the sealed envelopes with the price offer that should, therefore, be kept separate from the technical offer. After studying the technical offers, the committee determines an ordinal ranking of all the technical offers, i.e., it establishes who is "the first, the second, the third, …, the last" on purely technical grounds. Then, technical offers are evaluated cardinally; in other words, they are attached a monetary value. Suppose for simplicity there are only two offers, say "A" and "B", and that the quality of "A" is considered higher than the quality of "B". Therefore, if both "A" and "B" were offered at the same price, "A" would be awarded the contract. What if "B" offered a price "P" while "A" offered a higher price "P + X"? If the value of X is sufficiently small, "A" would still be preferred to "B" as its better technical performance more than compensates the more expensive price. However, as X is increased, there will be a value X = X* such that "A" and "B" are equivalent, that is, X* is the *monetary equivalent* that makes the buyer indifferent between "A" and "B". X* provides a monetary measure of the quality differential of the two offers. Given her preferences, the buyer can determine her monetary equivalent for any technical attribute offered by a bidder. At this stage, the commission would

consider the price bids. If the price offered by "A" turns out to be more expensive than the price offered by "B" + X*, then "B" would win, otherwise "A" would get the contract.

2.4 PREDICTABILITY OF SCORING RULES AND COLLUSION: IS THERE A RELATIONSHIP?

Simplicity and predictability are widely recognized to be good properties of procurement auction rules, since the buyer's preferences are revealed to the market and participants are able to better define their bidding strategies. These objectives are met when the relationship between the price offered and the score attained is made explicit to the participants, that is, when suppliers know *ex-ante* the MVP. Example 5 showed some adverse effects arising from the uncertainty about the points associated with bids: bidders could only base their bidding strategies upon expectations. Uncertainty can make worse off both the buyer (less savings) and bidders (risk of losing the auction).

One particular concern about predictability, though, is related to collusion. In general, transparent processes tend to favour collusive behaviours, since cartels are more likely to be sustainable. The general argument is that if procurement rules are predictable, the cartel can always observe its members' actions and punish any deviant from collusive behaviour. Therefore, more predictability may help sustaining the stability of collusive agreements. This is a concern for the procurer. However it is not clear whether and how the degree of predictability specifically related to scoring rules can have collusive effects, given that collusion is crucially affected by many other aspects of the auction which are not treated in this paper and that have very little to do with scoring rules (e.g., the number of lots, market structure, etc.).

We present a simple case to discuss the relationship (if any) between predictability in scoring rules and collusion. We imagine a procurement auction where participating firms agree to collude and that in force of the agreement one participant wins the lot and rewards the others for not competing. We compare two alternative scoring rules with different degree of predictability, namely with and without a priori knowledge of the MVP. An extremely simple and predictable scoring rule we can think of is the one that assigns the points proportionally to price reductions: *linear scoring*. In contrast, a less predictable rule is the *lowest bid scoring* where the score of all bidders depends on the lowest bid: for instance, "(lowest bid)/ (bidder's "i" bid)"[6]. What can we say about the collusive effects of these two scoring rules? Can we argue that the former gives more incentives to collude than the latter because of its higher predictability?

Imagine a scoring auction for a laptop, with a reserve price of €1000. N bidders participate to the auction and collude as follows: one of the N bidders, say "1", wins the contract bidding €999 and all other N-1 bid the reserve price. Surplus is then shared. Moreover, all participants agree to offer the same quality of laptop (e.g. the same screen dimensions, ram, processor, etc.), so that they get the same technical points. Collusion concerns price. With either linear scoring or lowest bid scoring, non-winning bidders can break the cartel by just offering €998. The cost of breaking the cartel is the same for all bidders and does not depend on the predictability of the scoring rule adopted: by reducing the price of €1, any bidder is effectively able to break the agreement and win the contract. The break of the cartel can occur under both scoring rules, with predictability playing no perceptible role. Therefore, in this simple case, the degree of predictability of the scoring rule has no effects on collusion.

[6] We provide more details about the properties of these scoring rules in section 3.

3. SCORING RULES FOR PRICE BIDS: SOME GENERAL FEATURES

In this section we analyse the properties of some price scoring rules that allow the buyer to transform price offers into a score. We focus on five scoring rules:

- **Linear scoring (Ls)**
- **Parabolic scoring (Ps)**
- **Lowest bid scoring (Lo)**
- **Highest bid scoring (Hs)**
- **Average scoring (As)**

We look at these scoring rules in terms of predictability, complexity and possible impact on competition. Before entering the details, we group the scoring rules into two families that have common features. These families are:

1. Simple scoring rules: the score of any bidder depends on his price bid only.
With this type of rules the relationship between the score, the price and the MVP are well known *a priori*. This property makes scoring rules belonging to this family predictable and simple. However, implementation is possible only if some parameters can be pre-defined (e.g., a reserve price is used). Linear scoring and parabolic scoring belong to this group.

2. Alternative scoring rules: the score of any bidder also depends on others bidder's price offers. In this family, the relationship between the score, the price and the MVP is *not* known ex-ante. Lowest bid, highest bid, average bid scoring belong to this group. These scoring rules are sensitive to the bid distribution. Within this family, the decision to accept/reject abnormally low tenders can influence the score achieved by all bidders and change the ranking.

3.1. SIMPLE SCORING RULES: LINEAR AND PARABOLIC SCORING

Linear Scoring (Ls). The linear scoring rule is a very simple way to transform price bids into a score. This rule is described by the following expression:

Price score = nn * [Reserve price – Price bid] / [Reserve price – Price threshold]

where "nn" is the maximum number of points (typically out of 100) available to bidders for price offers. The remaining (100 - "nn") points are attached to technical aspects. The reserve price is the price above which bidders get no points. The price threshold is a lower bound: the buyer cannot improve his score with further price reductions[7]. In absence of the price threshold (price threshold = 0), this scoring rule becomes a very simple linear rule which awards the maximum score "nn" only if the good is offered for free (a price discount of 100%).

The linear scoring rule has the following features:

- the score assigned to a single price bid does not depend on the bids distribution. This allows all participants to know *a priori* the score associated to their bid;

[7] The price threshold can be an important tool for the buyer to manage the intensity of price competition and to prevent abnormally low tenders (see below).

- it allows to know *a priori* the MVP,
- the MVP is constant;
- the reserve price must be pre-defined;
- the score assigned to a price offer is not sensitive to the rejection of abnormally low tenders;
- a price threshold can be introduced and tuned to optimize price competition;
- the maximum score ("nn") is achieved for a price offer equal to the price threshold; the minimum score of "0" is achieved for a price offer equal to the reserve price.

Because of these features, the linear scoring rule is very simple and predictable. As already shown above, simplicity and predictability allow both the buyer and the participants to better define their strategies. The MVP is obtained directly from the previous formula:

$$\text{MVP} = [\text{Reserve price} - \text{Price threshold}]/\text{nn}.$$

Example 6. Consider the procurement of a good with a reserve price of €1000, no threshold, and 70 points reserved to price offers. The MVP in this case is $1000/70 \approx €14.3$. In order to get 20 points, a bidder has to offer a discount of approximately €286 below the reserve price.

Notice that the higher the MVP, the more costly it is to get a point with a price reduction, thus making a bidder more reluctant to concentrate his bidding strategy on price reductions and opting instead on quality improvements. Conversely, a small MVP implies that a price discount can guarantee many points, thus making bidders more aggressive in their price bidding. From the above formula it is then clear that aggressive price bidding is more likely the higher "nn", the lower the reserve price, the higher the price threshold. We expand below on the latter, but similar arguments would also apply to the role of "nn" and of the reserve price.

Figure 1 shows the shape of two hypothetic linear scoring rules, one without price threshold and another one with price threshold equal to 50% of the reserve price. In this example, we assumed again a reserve price of €1000, with 70 and 30 points attached to price and technical aspects, respectively. It is straightforward to notice that, other things equal, the introduction of a threshold *increases* the slope of scoring rule and lowers the MVP. The slope has a special meaning: it measures how the score increases as price declines. In other words, the slope provides a measure of the *incentive for participants to bid on price*. The lower the MVP, the steeper the scoring rule, the higher the incentive to bid on price.

Practical Conclusion 5

Linear scoring is particularly appropriate for procurement auctions. Once you set the reserve price and the maximum points available, you can use the price threshold to fine - tune the intensity of price competition.

Notice that the price threshold splits the scoring line in two regions with different levels of price competition. Prices ranging between zero and the threshold involve no price competition since the maximum score is achieved when the price offered is equal to the threshold. Instead, intense competition arises in the interval of prices between the threshold and the reserve price. Therefore, the higher the price threshold, the more intense competition price is, but the smaller the region

involving such a competition. Thus the buyer has to be careful to avoid selecting a threshold level "too high" that can eliminate most competitive bids.

Figure 1 here

Parabolic scoring (Ps). Ps can be thought as an extension of linear scoring in that the score increases but at decreasing rates with further price reductions. Parabolic scoring takes the following expression when the price threshold is equal to 0:

$$\text{Price Score} = nn * [1 - (\text{Price bid} / \text{Reserve price})^2]$$

This is the simplest way to describe Ps. It is also possible to introduce a price threshold by slightly re-writing the formula[8].

This rule has the same properties of the linear scoring, except that:

- the MVP is non-constant, more precisely it is decreasing in price. This makes the rule more complicated compared to the linear one. Suppliers need to calculate the MVP for any price bid and define their bidding strategy accordingly, while in linear scoring the MVP is fixed and independent of any other factor;
- the rule stimulates aggressive bidding for prices close to the reserve price. As the price offered decreases, the price score increases as well but at a decreasing rate. Thus the incentive for additional discounts decreases (the curve tends to be flat and the MVP becomes relatively larger);

These features allow the buyer to fine-tune price competition by giving only mild rewards to discounts approaching the price threshold, thus progressively favouring bidding on technical improvements.

Practical Conclusion 6

Favour parabolic scoring when too aggressive price bids are likely to make quality to be neglected

Figure 2 shows an example of parabolic scoring with price threshold equal to 70% of the reserve price. The scoring line shows that there are no incentives to reduce bids below €30 as under linear scoring with the same threshold. However, incentives still remain low even below €50, thus making additional discounts unlikely to occur.

Figure 2 here

3.2. ALTERNATIVE SCORING RULES

[8] The standard equation of the parabolic scoring rule is: *Price score = nn*[a*(Price bid)^2 + b*(Price bid) + c]*, where the parameters "a", "b", and c" are appropriately set in order to obtain a scoring rule consistent to the chosen price threshold.

The scoring rules we treat here are more complex and less predictable compared to linear and parabolic scoring rules. These rules prevent bidders from knowing *a priori* the exact relationship between the price offered and the score, which, in turn, implies uncertainty about the MVP. However, the linear and parabolic scoring rules need an "anchor" represented by the reserve price. In some circumstances, this may not be easy to set. For instance, the buyer may be uninformed about the cost/value of procuring a complex project, therefore it may result optimal not fixing the reserve price and allowing market forces to reveal it. The reserve price may also be inappropriate when the market value of the good or service to procure is volatile (e.g., energy supply), since the price prevailing in the auction can differ much compared to the one used to construct the reserve price in the earlier phase of the auction design. In the following we review some scoring rules which do not need an *ex-ante* setting of parameters, except for the weight "nn".

Lowest bid scoring (Lo). Lo takes the following general expression:

Price score = nn * [Lowest bid / Price bid]

This scoring rule has the following features:

- the difference between the highest score and the lowest one is always lower than the maximum score.
- the score assigned to each bidder is crucially linked to the best price offer;
- if the best price offer is found to be abnormally low and rejected, the new ranking may change in unpredictable ways with little resemblance to the original ranking;
- this scoring rule can induce "aggressive" price bidding because participants by bidding low increase the likelihood to get the highest score reducing at the same time the score obtained by all other competitors.

Highest bid-lowest bid scoring (Hs). Hs takes the following general expression:

Price score = nn * [Highest bid - Price bid] / [Highest bid - Lowest bid]

This scoring rule has the following main features:

- the score assigned to each price offer depends on both the lowest and the highest price offers;
- it is "aggressive": regardless of the price distribution, the lowest price offer is assigned the maximum score, while the highest bid is assigned zero points.

Hs is independent of the *whole* bid distribution, i.e., its slope and the position are only determined by the highest and the lowest bid, while all bids in between are irrelevant. Hs assigns the maximum possible score differential between the lowest and the highest bid, scoring all other bids proportionally. This is indeed a straight line connecting the lowest and the highest bid.

Average scoring (As). As can take the following general expression:

If bid < Average bid ➔ Price score = nn

otherwise:

Price score = nn* [Highest bid – Price bid] / [Highest bid – Average bid]

This scoring rule has the following characteristics:

- when the price offered decreases, the score grows linearly up to its highest level as soon as the price offer equals the average bid;
- all bids below the average bid obtain the maximum score;
- particularly low prices are not rewarded in terms of score with respect to the other best offers, since they get the same score of the tender which is just below average;
- the score assigned to any price offer depends on all the price offers submitted.

If the market value of the good/service is unknown and volatile this scoring rule ensures that the contract is awarded to the average bid, which may proxy the true market value of the good/service in that moment. This rule discourages low tenders since no additional score is obtained bidding below the average. As have been used in several countries, as Italy, Peru, Taiwan, to deal with abnormally low tenders. However as should not be used for that scope, since the attempt to prevent low tenders may come at the considerable of costs of high prices. Lower-than average prices do not increase chance to win the auction. Therefore, bidders will not be encouraged to submit their lowest price, rather, they will try to forecast the average and bid that value. This clears out the price competition in the auction and can lead to significant loss of savings for the procurer. Buyers should be aware of such adverse effects when using As and other similar scoring rules.

Practical Conclusion 7

When the market value of goods/services to procure is volatile and a reserve price is difficult to set, you can use with caution one of the alternative scoring rules Lo and Hs

3.3. SCENARIO ANALYSIS

We now compare the above scoring rules on the basis of different assumptions about the price bid distribution. This comparative analysis attempts to highlight how under different bid distributions these scoring rules behave in terms of: 1) the score achieved by bidders, 2) the score differential among bidders, 3) the incentives to compete on price, and 4) the final ranking given the technical points. Notice that the analysis does *not* take into account that bids depend on the scoring rule. Indeed, different scoring rules provide participants with different incentive structures. Therefore we do expect bid distributions to somewhat vary according to the scoring rule adopted. This element is not considered here. Nevertheless, this kind of comparative analysis allows the buyer to gather useful information for the choice and the set-up of the scoring rule, since it shows how bidders' incentives to compete on price vary in response to both different scoring rule settings and bids

distributions. This information can help the buyer to optimally balance, according to her preferences, the potentially conflicting goals of high quality and low price. With this type of exercise the buyer can understand how much discount is necessary for a "cheap"- low quality offer to defeat a "more expensive"- higher quality offer.

We focus on two possible bid distributions, which are representative of two alternative kinds of markets:

1. **Uniform distribution (asymmetric bidders)**. Suppliers' price bids are uniformly distributed along the price line. This distribution is characterized by a large variance of bids indicating that bidders are heterogeneous with respect to some relevant characteristics. For instance, bidders may experience significant differences in cost structures.
2. **Concentrated distribution and a very low tender (all symmetric bidders except one):** Suppliers' price bids are concentrated around their mean value, except one very low price bid. In this context, bidders are substantially symmetric, but there is one "maverick" bidder posting a very aggressive bid that may corresponded to an abnormally low tender (ALT).

We assume the reserve price, the maximum number of points awarded to price offers ("nn"), and the threshold price to be equal to €1,000,000, 30 and €0 respectively, for both linear and parabolic scoring rules.

1. Uniform distribution:

Table 2 here
Figure 3 here

In comparing the scoring rules we find the following:

- With As, Hs and Ls the score grows linearly as the price decreases; in As the score grows up to the average bid, while in Hs the score grows up to the lowest bid and in Ls it grows up to the zero price bid;
- Lo and Ps are non linear;
- As and Hs assign all the score differential available; As assigns a score of zero to the highest bid, and the highest score to all bids equal or below the average bid; Hs assigns a score of zero to the highest bid, and assigns the maximum score only to the lowest bid;
- Lo, Ls and Ps do not assign all the score differentials available. In general Lo never assigns all the score differential because it always assigns a strictly positive score to the highest bid, whereas Ls and Ps assign all the score differential only when the highest bid equals the reserve price and the lowest bid equals the price threshold;
- As provides bidders the highest incentives to bid on price since the increase of the score in response to a unitary reduction of price is higher with respect to all other rules (As is the steepest for prices above the average price offer). In contrast, when price offers are very low, Lo provides the highest incentives. Rules Hs and Ls provide the similar incentives to compete on price in this example.

With a uniform bid distribution, these scoring rules are quite heterogeneous in the incentive they provide on price competition. Therefore, when bidders are asymmetric, incentives to compete on price may vary significantly according to the scoring rule adopted.

2. Concentrated distribution on high prices and one abnormal low tender:

Table 3 here
Figure 4 here

When the bid distribution is concentrated on high prices but there is one abnormal low tender, we have that:

- As, Lo and Hs still assign the maximum score available, whereas Ps and Ls respectively assign slightly less than the maximum score and about 2/3 of it;
- the score differential between the abnormally low tender and the second best bid is zero with As, it is similar with Ls, Ps and Lo, namely slightly above the half of the points available, and it is very large with Hs;
- the abnormal low tender does not change score differentials among all the other bids under Ls and Ps, while it affects them under the alternative scoring rules, especially with Hs;
- the scoring rules face quite similar slopes except for As, which is very steep relative to the others.

3.4. PRICE SCORING, BEHAVIOURS AND INCENTIVES

The incentive to bid on price changes under different scoring rules. For instance, consider what happens in Figure 4. Here, As assigns the smallest number of points when suppliers offer a price of €900. However, as clearly indicated in the figure, around this price the slope of As is greater than the slope of any other scoring rule. The steeper the scoring rule, the bigger the incentive for an additional unitary discount of price. Figure 4 indicates that, even though As assigns the lowest score at price €900 it provides suppliers the strongest incentive to bid on price. Therefore, depending on the scoring rule, incentives to compete on price and thus price bids effectively submitted can be rather different: other things equal, we expect lower prices from "competitive" scoring rules.

In table 4 we report a summary of the results emerged in the relative comparison of the scoring rules. Each scoring rule is compared on the basis of four features: simplicity, predictability, competition and sensitivity to bid distribution. The scale of evaluation ranges from 0 to 3. Ls and Ps, that we recommend for procurement auctions, are the simplest and most predictable, mainly because not sensitive to bid distribution, but they appear to be less competitive with respect to Lo and Hs. However, price competition can be stimulated by appropriately using the price threshold and the maximum number of points ("nn").

4. CONCLUDING REMARKS

Procurement contracts are often awarded taking into account attributes other than just price. Awarding these contracts involves the use of a scoring auction and the definition of a precise scoring rule. Scoring rules allow the buyer to find the appropriate balance between the price and the non-price attributes (quality) according to her preferences.

This paper has dealt with the choice and design of scoring rules. We have discussed how the buyer can set the scoring rule to solve the typical price-quality tension arising in any procurement auction. One effective way to do it is to define a simple and predictable linear scoring rule. We provided some Practical Conclusions to compare various common types of scoring rules. Scenario analysis can help to understand which scoring rule might perform better in what context.

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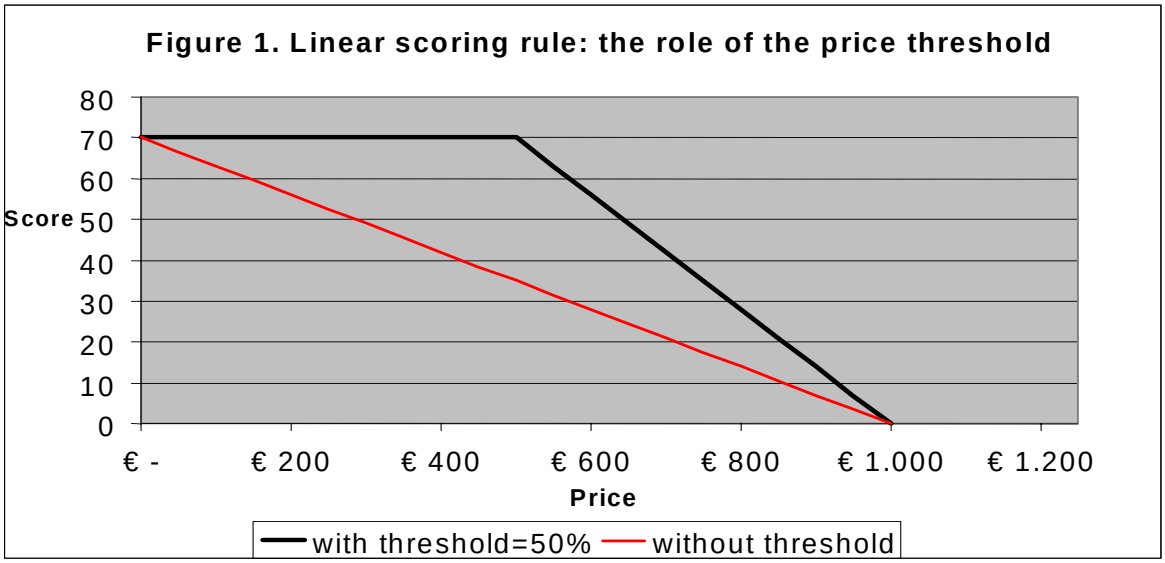
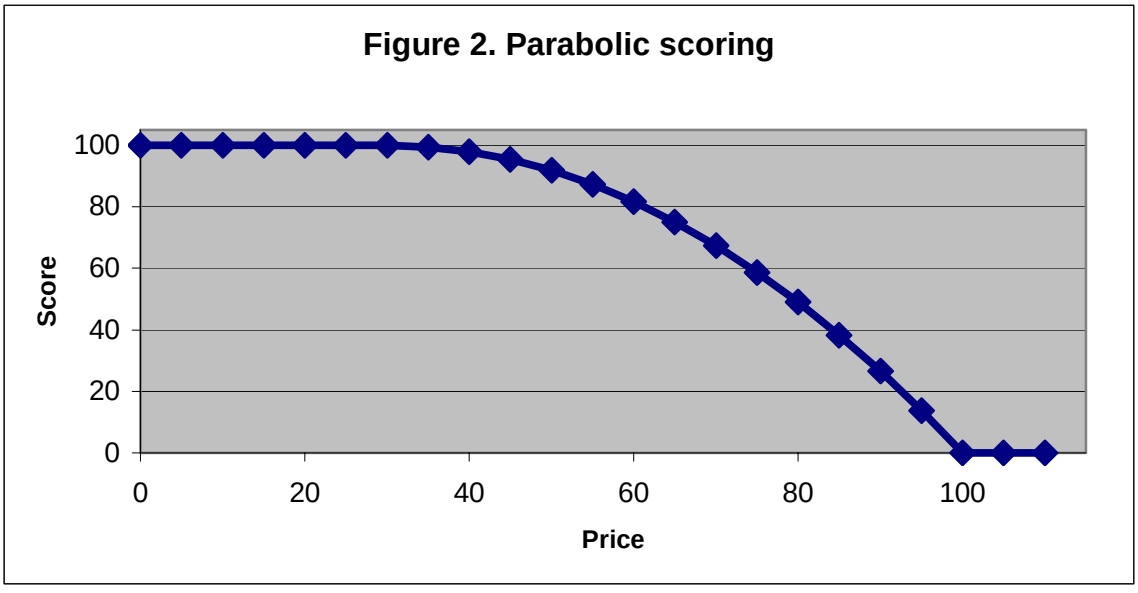
TABLES AND FIGURES





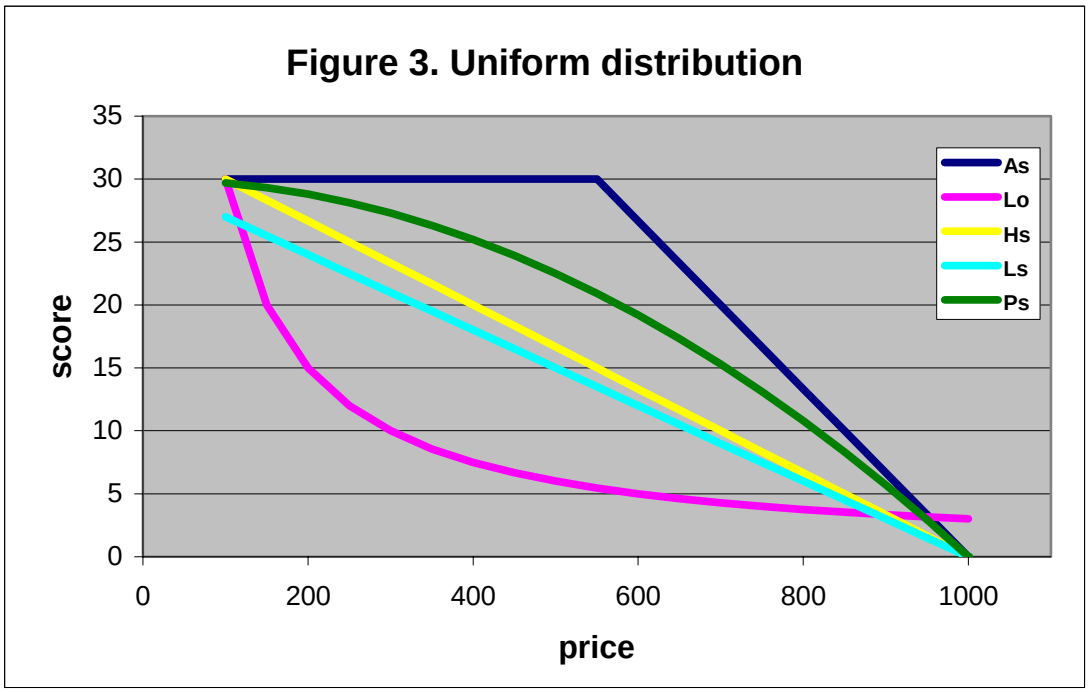


Figure 3. Uniform distribution
35
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score
0
200
400
600
800
1000
price
As
Lo
Hs
Ls
Ps

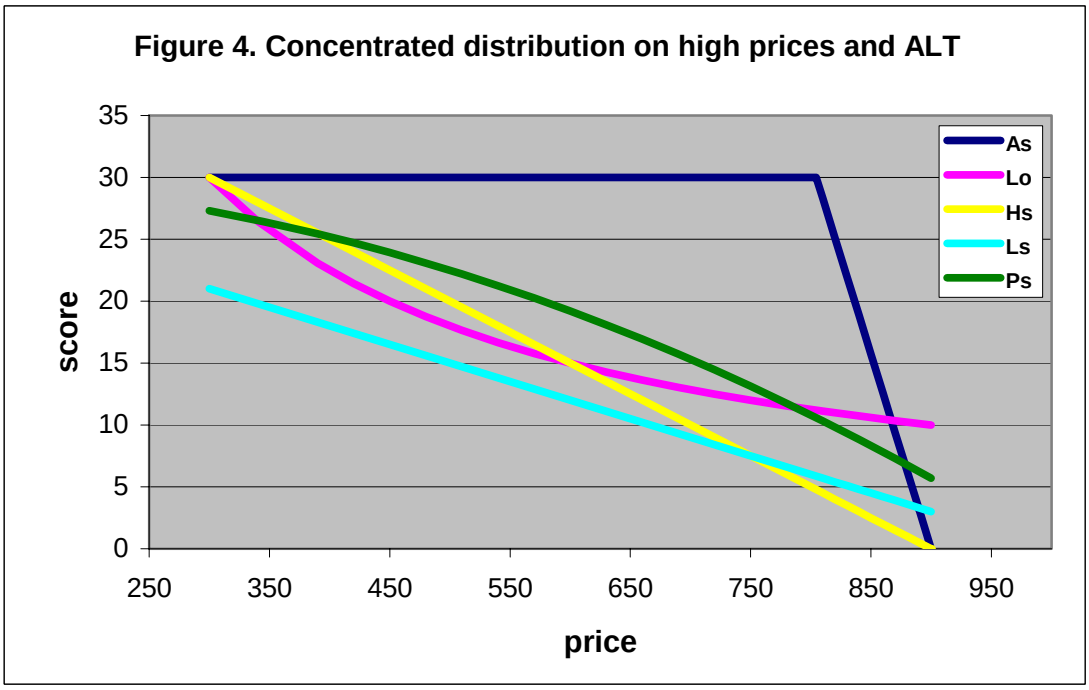
Figure 4. Concentrated distribution on high prices and ALT
35
30
25
20
15
10
5
0
score
250
350
450
550
650
750
850
950
price
As
Lo
Hs
Ls
Ps

Table 1

Price		Non-price attribute	
Bid	Points	Monitor	Points
€ 5,000	0	14"	0
€ 4,500	10	15"	10
€ 4,000	20	16"	20
€ 3,500	30	17"	30
€ 3,000	40		
€ 2,500	50		
€ 2,000	60		
€ 1,500	70		
Max points = 70 + 30 = 100			

Table 2 - Uniform distribution

€ 1,000	0.0	3.0	0.0	0.0	0.0
€ 900	6.6	3.3	3.3	3.0	5.7
€ 800	13.3	3.7	6.6	6.0	10.8
€ 700	20.0	4.2	10.0	9.0	15.3
€ 600	26.6	5.0	13.3	12.0	19.2
€ 500	30.0	6.0	16.6	15.0	22.5
€ 400	30.0	7.5	20.0	18.0	25.2
€ 300	30.0	10.0	23.3	21.0	27.3
€ 200	30.0	15.0	26.6	24.0	28.8
€ 100	30.0	30.0	30.0	27.0	29.7
Δ points (best bid - worst bid)	**30.0**	**27.0**	**30.0**	**27.0**	**29.7**
Δ points (best bid - 2nd best bid)	**0.0**	**15.0**	**3.3**	**3.0**	**0.9**

Table 3 – Concentrated distribution on high prices and an abnormal low tender

Bids	Score				
	As	Lo	Hs	Ls	Ps
€ 900	0.00	10	0.0	3.0	5.7
€ 890	3.1	10.1	0.5	3.3	6.2
€ 880	6.2	10.2	1.0	3.6	6.7
€ 870	9.4	10.3	1.5	3.9	7.2
€ 860	12.5	10.4	2.0	4.2	7.8
€ 850	15.7	10.5	2.5	4.5	8.3
€ 840	18.8	10.7	3.0	4.8	8.8
€ 830	22.0	10.8	3.5	5.1	9.3
€ 820	25.1	10.9	4.0	5.4	9.8
€ 810	28.2	11.1	4.5	5.7	10.3
€ 300	30.0	30.0	30.0	21.0	27.3
Δ points (best bid - worst bid)	**30.0**	**20.0**	**30.0**	**18.0**	**21.6**
Δ points (best bid - 2nd best bid)	**1.8**	**18.9**	**25.5**	**15.3**	**17.-0**

Table 4

Comparing scoring rules on the basis of some key features					
Feature	**Scoring rule**				
	As	**Lo**	**Hs**	**Ls**	**Ps**
Simplicity	0	3	1	3	2
Predictability	0	0	1	3	3
Competition	0	3	3	1	2
Sensitivity to bid distribution	2	3	1	0	0
0 minimum level, 3 maximum level					

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