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First-Price Bidding and Entry Behavior in a Sequential Procurement Auction Model

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Abstract. We introduce a procurement auction model where capacity-constrained firms face a sequence of two procurement auctions, each of them in the first-price sealed-bid design. Our main findings are that the firms' entry decisions depend on relative project completion cost levels and that equilibrium bidding in both auction stages deviates from the standard Symmetric Independent Private Value auction model (SIPV) due to opportunity costs of bidding created by possibly employed capacity. The model highlights the fact that firms with identical completion costs for the first project may differ in entry and bidding strategies. In addition, experimental data is reported in order to assess the predictive power of the model.

1 Introduction

Theoretical contributions to the analysis of procurement auctions tend to focus on the allocation of single projects ignoring outside options of firms, e.g. [14], [10], [4], [2], [3]. However, recent empirical research on repeated procurement auctions suggests that outside options in the sense of additional future procurement auctions of similar projects affect firms' entry and bidding behavior in real life, see [5] and [7]. E.g., the study [7] finds that a firm which did not win a highway procurement contract earlier in a sequence of auctions run by the Californian Department of Transportation (1994-1998) is twice as likely to enter a subsequent auction than a firm which already won a (large) contract. This evidence suggests that firms are aware of their opportunity costs of bidding created by employed capacity. Thus, firms seem to be choosy with respect to entry if facing an auction sequence of non-identical procurement contracts and might include these opportunity costs in their submitted bids.

This paper studies this kind of entry decision in the context of privately known completion cost rankings and analyzes how firms refine their bidding strategies with opportunity costs of early bid submission. Our main theoretical findings are that the entry decision depends on relative project completion cost levels and equilibrium bidding in both auction stages deviates from the standard SIPV model and its sequential formulation with homogenous goods due to opportunity costs of bidding created by possibly employed capacity. It follows from the analysis that firms with identical completion costs for the first project may differ in entry and bidding strategies.

In addition, this paper presents experimental evidence for our sequential procurement auction model that suggests that its theoretical implications might explain observed patterns of entry behavior in repeated real-life procurement auctions. In turn, this points to an explicit role for the auction environment in empirical studies on static procurement auctions.

2 Model and Equilibrium Behavior

We consider two risk-neutral firms with capacity to complete a single project due to capacity constraints. Completion costs for two subsequently auctioned projects, L and M, are private information to each firm. It is common knowledge that firm i's costs of completion are jointly drawn from $f(l_i,m_i)$ with domain $[\underline{c},\overline{c}]$ and stochastic equivalent in the sense of $f(l,m) = f(m,l)$ for every $(l,m) \in [\underline{c},\overline{c}]^2$ implying $E[L_i] = E[M_i]$. It follows that each firm faces either lower completion cost for project L or project M. If cost realizations of a firm are such that $l < m$, this firm is said to have a cost advantage for project L, the reversed inequality indicates a cost advantage for project M. Although completion costs of a single firm may be correlated across projects, pairs of completion costs of different firms are independently distributed.

In each procurement auction, a participating firm may submit a sealed bid where the lowest bid wins the project and the amount bid is paid in exchange for completion of the project. However, bids cannot exceed maximum completion costs $\overline{c}$ which may be interpreted as the procurers outside option, otherwise infinite prices result. We assume that the auctioneer cannot set a price below maximum completion costs $\overline{c}$ and that resale of projects is not feasible. If there happens to be a bidding tie, any auctioneer employs a fair chance mechanism to break it. The sequence of auctions begins with the procurement auction of project L where the winner is announced before project M is auctioned off. Thus with two firms, any firm can infer if it faces competition in auction M before it submits its bid.

In addition to equilibrium bidding functions for each auction stage, b^L and b^M, a firm's strategy also includes a decision to submit a bid in the first auction or skip bidding for project L. In general, firm 1 participates in auction L if its expected profit from bidding exceeds the opportunity cost arising from possibly being excluded from bidding for project M, formally

$$E[\Pi_1^{L+M}|(l_1,m_1)] \geq E[\Pi_1^{M}|(l_1,m_1)],$$

where $E[\Pi_1^{L+M}|(l_1,m_1)]$ denotes firm 1's expected profit if it bids in the procurement auction for project L and - if unsuccessful - continues bidding in auction M and $E[\Pi_1^{M}|(l_1,m_1)]$ is its expected profit if it skips the first auction and bids only for the subsequently auctioned project M.

The firm's decision to submit a bid in auction L depends on the relationship of its project completion costs. In order to formalize the entry decision, we introduce the critical-value function $g_1 : [\underline{c},\overline{c}] \to [\underline{c},\overline{c}]$ which specifies a cut-off point for completion costs of project L, depending on completion costs of project M, beyond which it is not worthwhile for firm 1 to participate in the auction of project L. Specifically, the decision rule to submit a bid for project L is given by entry function $\varepsilon(\cdot)$ as follows:

$$\varepsilon_1(l_1, m_1) = \begin{cases} \text{Enter Auction } L & \text{if } l_1 \leq g_1(m_1) \\ \text{Skip Auction } L & \text{if } l_1 > g_1(m_1) \end{cases}$$

where the critical-value function $l_1^{crit} = g_1(m_1)$ is implicitly defined by the equality of expected profits from entering auction L and corresponding opportunity cost:

$$E\left[\Pi_1^{L+M} \middle| \left(l_1^{crit}, m_1\right)\right] = E\left[\Pi_1^{M} \middle| \left(l_1^{crit}, m_1\right)\right]. \tag{1}$$

By definition, firm 1 is indifferent between entering auction L and skipping it if its completion cost pair satisfies $l_1^{crit} = g_1(m_1)$. In symmetric equilibrium, equation (1) is given by (for details see [12]):

$$\begin{aligned} 0 = \frac{\overline{c} - m}{2} + \frac{2\overline{c} - [g(m) + m]}{2} \left[\frac{1}{2} - \int_{\underline{c}}^{\overline{c}} \int_{g(y)}^{\overline{c}} f(x,y)\, dx\, dy\right] \\ + [\overline{c} - g(m)] \int_{\underline{c}}^{\overline{c}} \int_{g(y)}^{\overline{c}} f(x,y)\, dx\, dy - (\overline{c} - m) \int_{\underline{c}}^{\overline{c}} \int_{\underline{c}}^{g(y)} f(x,y)\, dx\, dy \\ - \int_{m}^{\overline{c}} \int_{g(y)}^{\overline{c}} (y - m) \cdot f(x,y)\, dx\, dy\,. \end{aligned}$$

The properties of the critical value function $g(m)$ in symmetric equilibria are summarized in the next proposition that is proved in [12].

Proposition 1. *For all symmetric perfect Bayesian equilibria characterized by the representative firm's strategy* $[b^L(l,m),\ b^M(m),\ \varepsilon(m)]$ *and the density function* $f(l,m)$*:*

(a) *There exists a (nonempty) compact and convex set of completion cost pairs where it is rational for a firm to bid for project* L *although it has a cost advantage for completing project* M*. This subset is defined by* $G = \{(l,m) \in [\underline{c}, \overline{c}] \mid m \leq l \leq g(m)\}$.

(b) The critical value function $g(m)$ exists and

- **(i)** $m < g(m) < \overline{c}$ f. $m \in [\underline{c}, \overline{c})$, $g(\overline{c}) = \overline{c}$, $g(\underline{c}) > \underline{c}$,
- **(ii)** $g'(m) > 0$,
- **(iii)** $g''(m) < 0$ f. $m \in [\underline{c}, \overline{c})$ and $g''(\overline{c}) = 0$.

It follows from proposition 1 that a firm always enters auction L if it faces a cost advantage for this project, i.e. $l \leq m$. Then it can earn at least $\overline{c} - m$. In contrast, a cost advantage for completing project M implies the impossibility of the firm to secure it the same return in auction L as it could earn in auction M being the only bidder. However, if the competing firm skipped auction L, too, then there is competition in the second auction with the risk of low or even zero profits due to aggressive bidding. Therefore a firm with lower cost for project M may wish to participate in the first auction and win the unloved project L at a high price to insure itself against low profits resulting from fierce competition in the second auction, although it actually prefers losing the auction for project L. Figure 1 illustrates these results for a representative firm with completion costs (l, m).

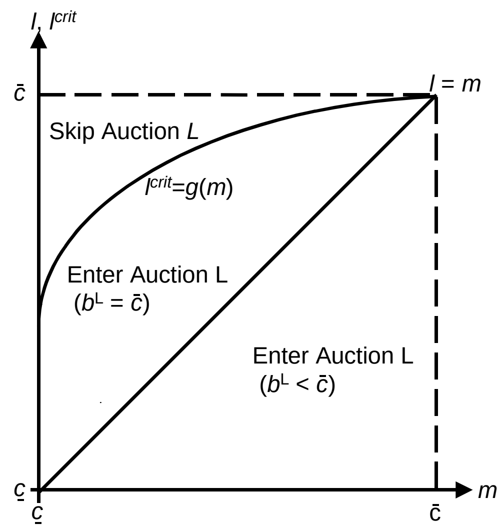

Fig. 1. A typical critical-value function $g(m)$. If $l \leq g(m)$, the firm submits a bid in the first auction. A firm with relatively high cost for completing the first project, i.e. $l > g(m)$, doesn't submit a bid.

The symmetric equilibrium bidding strategy in a one-shot first-price auction is well-known. For a discussion see e.g. [9] and [16]. However, in our dynamic setting additional issues arise. Consider first the auction for project L. Any of the two firms participating in the first auction anticipates that if it doesn't win project L, it remains the only bidder in the subsequent auction M and receives $\bar{c} - m$. Thus it may submit a very large bid for project L since it is, at least partially, insured against losing auction L. A firm with a cost advantage for project M knows that it cannot achieve such a high return in auction L as it would receive by winning the second auction after it lost the first one. Thus, provided the firm decides to participate in auction L, it minimizes chances of winning project L by submitting the highest feasible bid. In contrast, if a firm has a cost advantage for project L, then it tops its completion costs l with its certain return from auction M and uses the revised cost parameter $\tilde{l} = l + \overline{c} - m$ in auction L.

If there is no bidding competition in the auction for project M, then any firm bidding for it will submit the maximum feasible bid to maximize profits. In case of bidding competition, the updated belief concerning the competitor's cost parameter for project M takes into account that it also skipped auction L which may not be equilibrium behavior for every type. The appropriate a posteriori pdf is denoted by $f_{M|\text{Skip}}(m)$ and gives the (equilibrium) density that a firm with completion cost realization m for project M bids only in auction M.

The equilibrium bidding functions are summarized in the next proposition, for a proof see [12].

Proposition 2. *Equilibrium Bidding Functions in Auctions L and M*
The equilibrium bidding functions of a firm with cost pair $(l, m) \in [\underline{c}, \overline{c}]^2$ are given by:

$$b^L(\tilde{l}) = \begin{cases} \overline{c} & \text{if } l \geq m \\ \tilde{l} + \frac{\int_{\tilde{l}}^{\overline{c}} [1 - F_{\tilde{L}}(x)] dx}{1 - F_{\tilde{L}}(\tilde{l})} & \text{otherwise} \end{cases}$$

if it submits a bid for project L where $F_{\tilde{L}}(x) = \int_{\underline{c}}^{x} \int_{\overline{c}+\underline{c}-\tilde{l}}^{\overline{c}} f(m - \overline{c} + \tilde{l}, m)\, dm\, d\tilde{l}$ with $x, \tilde{l} \in [\underline{c}, \overline{c}]$ and $\tilde{l} = l + \overline{c} - m$, and

$$b^M(m) = \begin{cases} \overline{c} & \text{if it is the only bidder} \\ m + \frac{\int_{m}^{\overline{c}} [1 - F_{M|Skip}(x)] dx}{1 - F_{M|Skip}(m)} & \text{otherwise} \end{cases}$$

if it submits a bid for project M where $g(x)$ denotes the competitor's critical-value function, $f_{M|Skip}(x) = \int_{g(x)}^{\overline{c}} f(y, x)\, dy \,/\, \int_{\underline{c}}^{\overline{c}} \int_{g(s)}^{\overline{c}} f(y, s)\, dy\, ds$ and $F_{M|Skip}(x) = \int_{\underline{c}}^{x} f_{M|Skip}(s)\, ds$.

3 Impact of Auction Environment on Bidding

In this section, we briefly demonstrate how bidding behavior in the first auction varies with the value that a firm places on the opportunity to participate in a second auction. This option value itself depends on a firm's completion cost for the second project. For the purpose of illustration, consider a specific example with two firms where completion costs are distributed uniformly: $(l, m) \sim U[20, 100]^2$.

(a) (Base Scenario): Suppose the completion cost for project M to be fixed at some level below maximum completion cost, say $m = 80$. The bidding function for the first project L is

$$b^L(l, m = 80) = \begin{cases} \frac{2}{3} \cdot \frac{l^3 + 30l^2 - 1{,}664{,}000}{l^2 - 12{,}800} & l < 80 \\ 100 & l \geq 80\ . \end{cases}$$

Figure 2 illustrates that bids in the dynamic auction environment with $m = 80$ substantially exceed bids in a one-shot auction determined by $b(l)$ given below.

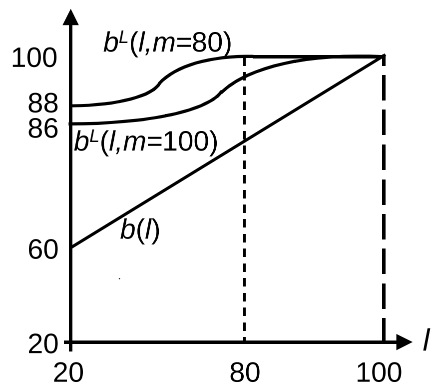

Fig. 2. Bidding for project L under various outside options

(b) (One-Shot Auction) There is no second auction such that the first auction reduces to the standard SIPV bidding function in a procurement context:

$$b(l) = 50 + 0.5l$$

(c) (Virtually No 2nd Auction): Consider base scenario (a) with maximum completion cost $m = 100$. Compared to $b^L(l, m = 80)$, the larger completion cost m shifts the bidding function $b^L(l, m = 100)$ downwards,

$$b^L(l, m = 100) = \frac{2}{3} \cdot \frac{l^3 - 30l^2 - 1,660,000}{l^2 - 12,400 - 40l} .$$

The exemplified change of $b^L(l, m)$ in response to increases in m illustrates typical comparative static behavior. Note that bids with virtually no second auction do not coincide with bids in the one-shot auction since completion costs are private information.

4 Experimental Results

The preceding section highlighted the fact that variations of the second round auction game change the equilibrium bidding function in the first round. In particular, larger costs for completing the second project, which correspond to a lower value of the second round game, lead to more aggressive bidding for the first project. This theoretical prediction and that on entry behavior are fulfilled in the laboratory implementation of our auction model. We sketch some of the results that are obtained by Brosig and Reiß in a study

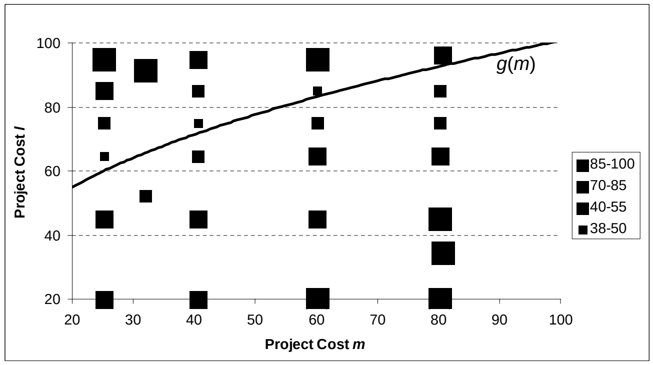

Fig. 3. Entry decisions in line with theory

of sequential procurement auctions at the Magdeburg Laboratory for Experimental Economics (MaxLab), see [1].

Figure 3 summarizes data on entry generated in one of the treatments where 24 subjects played the procurement auction sequence, each of them with 28 pairs of completion costs. Each marking in the $(m,\ l)$-space represents one completion cost pair. The size of the markings indicates the frequency of decisions on entry in line with the theoretical prediction relative to all observed decisions on entry for that cost pair. Out of a total of 672 observed entry decisions, approximately 70% are correctly predicted by our theory. It

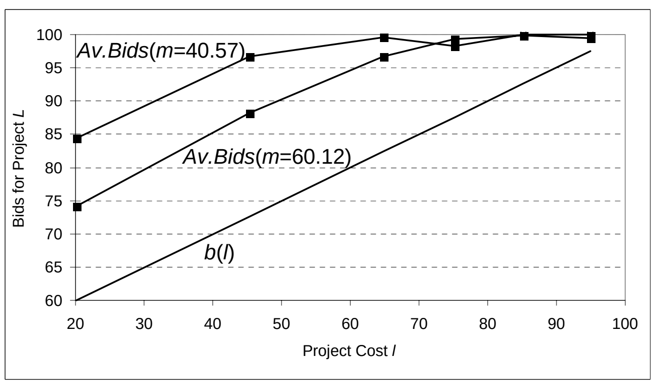

Fig. 4. Observed bidding for project L

is apparent from figure 3 that the number of correctly predicted entries rises as the cost pair is farther away from the critical-value function. In the vicinity of $g(m)$, a false entry decision is less costly and on its graph, the cost is zero by definition. Thus, the theory's predictive power increases as the expected cost of incorrect entry is larger. This is rigorously confirmed in [1].

Figure 4 depicts average bids for project L as a function of l for two different cost levels for the second project. Apparently, observed bids for m=40.57 (significantly) exceed those for m=60.12 in the relevant range 20–60.12 which is in line with our sequential procurement auction model, see section 3. Both bid samples (significantly) exceed predictions of the standard one-shot auction model. Since the overbidding phenomenon in standard auction experiments translates to underbidding in the procurement auction context, this fact is particularly remarkable and strengthens the suggestion that subjects included opportunity costs of early bidding in their bids for project L.

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