

Entry and Bidding Behavior in Sequential Procurement Auctions*

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Abstract

We analyze entry and bidding behavior of capacity-constrained firms in a sequence of two procurement auctions where lowest sealed bids win. In the model, firms with a cost advantage in completing the project auctioned off at the end of the sequence may enter the unfavored first auction hoping to lose it. Equilibrium bidding in the first auction deviates from the standard Symmetric Independent Private Value auction model (SIPV) due to opportunity costs of bidding created by possibly employed capacity.

Revenue equivalence between the first-price and second-price sealed-bid auction formats suggests that these results on entry and bidding don't depend on the auction design.

The model explains that firms with identical completion costs for the first project may differ in entry and bidding strategies.

Keywords: sequential auctions, procurement auction, auction entry

JEL classification: C72, D44, L51

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1 Introduction

A key feature of procurement auctions is its sequential nature. Usually procurers differ across regions, countries, firms, and institutions and set independently from each other different auction dates implying a sequence of auctions. Sometimes projects to be auctioned off differ substantially from each other, but often they are similar and, moreover, offered for execution during the same window of time. Although the sequential nature of procurement auctions is prevalent, most theoretical studies implicitly abstract it away by focussing on a single procurement auction in isolation (e.g. Riordan and Sappington (1987), McAfee and McMillan (1987b), Budde and Göx (1999), Dasgupta and Spulber (1989), Rob (1986), Celentani and Ganuza (2002)).

In the few contributions to the sequential procurement auction theory, it is common to assume that any bidding firm has unlimited capacity to execute all sequentially offered projects, see e.g. Gale et al. (2000) and Luton and McAfee (1986).¹ However, empirical studies on sequential procurement auctions point to the relevance of capacity constraints. De Silva et al. (2002) find that bids are positively correlated with firms' capacity. Jofre-Bonet and Pesendorfer (2000, 2003) report that a firm which didn't win a highway procurement contract earlier in a sequence of auctions is twice as likely to enter a subsequent auction than a firm which already won a (large) contract. This evidence suggests that firms are aware of their opportunity costs of bidding created by employed capacity and might be choosy if facing an auction sequence of non-identical procurement contracts.

Our paper differs from the literature in that firms are capacity-constrained, procurement auctions are in the first-price sealed-bid design, and projects are stochastically equivalent. Since a potential bidding firm finds itself restricted to execution of a subset of sequentially offered heterogenous projects, it must decide which procurement auctions to enter entailing a selection of projects it wishes to possibly end up with.

For a procurement sequence of identical projects, the analysis of Weber (1983) suggests that equilibrium expected payments of winning bidders coincide and, hence, any firm may want to begin bidding at the start of the sequence. If, however, firms prefer to execute one project to another due to different project completion costs, it is a priori not clear if a firm with more favorable completion costs for projects to be auctioned later in the sequence submit bids for projects auctioned earlier.

This paper studies this kind of entry decision and analyzes how firms refine their bidding strategies with opportunity costs of early bid submission. Our main findings are that the entry decision depends on *relative* project completion cost levels and equilibrium bidding in both auction stages deviates from the standard Symmetric Independent Private Value auction model

¹An important exception is Elmaghraby (2003).

(SIPV) and its sequential formulation with homogenous goods and single-unit demand. Firms with lower completion costs for the first project auctioned off always submit bids while firms with lower completion costs for the project subsequently auctioned off only participate if their opportunity costs are not too large. Each firm entering the first auction includes its option value of the second project in its bid for the first project. Moreover, we show that the first-price and second-price auction designs generate identical equilibrium entry strategies and identical expected payments for procurers suggesting that our results are independent of the choice of the particular auction stage format.

The next section introduces our model and its symmetric equilibrium. In section 3, the equivalence of first- and second-price auctions is demonstrated. Section 4 provides a parameterized example. The impact of alternative transaction opportunities on bidding behavior is analyzed in section 5 and section 6 concludes.

2 The Model

There are two risk-neutral firms, each endowed with capacity to complete a single project.² Two projects, L and M , are sequentially auctioned off. Subcontracting is prohibitively costly.³ The firms' cost of completing any of the two projects are their private information. In order to formalize the similarity of both projects and the aspect that the ranking of completion costs is unknown⁴ to competitors, we assume, in the spirit of Gale and Hausch (1994), that projects are stochastically equivalent. In particular, it is common knowledge that firm i 's costs of completion are jointly drawn from $f(l_i, m_i)$ with domain $[\underline{c}, \bar{c}]^2$ and stochastically equivalent in the sense of $f(l, m) = f(m, l)$ for every $(l, m) \in [\underline{c}, \bar{c}]^2$ implying $E[L_i] = E[M_i]$. Although completion costs of a single firm may be correlated across projects, pairs of completion costs of different firms are independently distributed. If cost realizations of firm i are such that $l_i < m_i$, this firm is said to have a cost advantage for project L , the reversed inequality indicates a cost advantage for project M .

²Although the restriction that firms are required to complete at most a single project seems severe, closer observation reveals that this element is common in procurement auctions. Firstly, procurers may stipulate exclusive project completion to avoid that its competitors running a similar project gain benefits through a contractor working for both procurers. Secondly, a firm may face capacity constraints if projects run simultaneously and require relatively large amounts of its resources. Finally, firms may voluntarily decide not to execute simultaneously several risky projects to prevent changes in the risk distribution of their entrepreneurial activities.

³Empirical studies on procurement bidding (e.g. Jofre-Bonet and Pesendorfer 2000, 2003) find that the probability that a firm participates in an auction and that a participating firm wins the auction decreases in its backlog. This points to the fact that firms regard subcontracting as costly and not always as a feasible option to weaken their capacity constraints.

⁴Unlike the second-price procurement auction model in Elmaghraby (2003) where it is assumed that the second project is always more costly than the first one.

In each procurement auction, a participating firm may submit a sealed bid where the lowest bid wins the project and the bidded amount is paid in exchange for completion of the project. However, bids cannot exceed maximum completion costs \bar{c} which may be interpreted as the procurers outside option. We assume that the auctioneer cannot set a price below maximum completion costs \bar{c} and that resale of projects is not feasible. If there happens to be a bidding tie, auctioneers employ a fair chance mechanism to break it. The sequence of auctions begins with the procurement auction of project L where the winner is announced before project M is auctioned off. Thus, with two firms, any firm knows if it faces competition in auction M before it submits its bid.

Since both firms are ex ante symmetric, we restrict attention to the case of symmetric equilibria. We refer to the representative firm as firm 1. In addition to equilibrium bidding functions for each auction stage, a firm's strategy also includes a decision to submit a bid in the first auction or skip bidding for project L . Intuitively, there must be a region of cost types where firms reject to bid in auction L since a firm with completion cost $l = \bar{c}$ cannot make any profit by completing this project and, moreover, is - if it has won project L - excluded from participating in auction M where its expected profits may be positive due to more favorable costs of completion $m < \bar{c}$. Obviously, these extreme cost pairs highlight that opportunity costs, which coincide with expected profits from skipping auction L , exceed expected profits from bidding for project L . In general, firm 1 participates in auction L if its expected profit from bidding exceeds opportunity costs arising from possibly being excluded from bidding for project M , formally

$$E[\Pi_1^{L+M} | (l_1, m_1)] \geq E[\Pi_1^M | m_1],$$

where $E[\Pi_1^{L+M} | (l_1, m_1)]$ denotes firm 1's expected profit if it bids in the procurement auction for project L and - if unsuccessful - continues bidding in auction M and $E[\Pi_1^M | m_1]$ is its expected profit if it skips the first auction and bids only for the subsequently auctioned project M . Profits are random since completion costs of any competitor are unknown and determine its bidding behavior.

The firm's decision to skip auction L depends on the relationship of its completion costs. In order to formalize the entry decision we introduce the entry indifference curve $g_1 : [\underline{c}, \bar{c}] \rightarrow [\underline{c}, \bar{c}]$ that assigns a level for the completion cost of project L to each cost level of project M such that the firm is indifferent between taking part in auction L and skipping it. The entry indifference curve $l_1^{crit} = g_1(m_1)$ is implicitly defined by the equality of expected profits from entering auction L and corresponding opportunity cost:

$$E \left[\Pi_1^{L+M} | (l_1^{crit}, m_1) \right] = E \left[\Pi_1^M | m_1 \right]. \quad (1)$$

Since it cannot be worthwhile for a firm to participate in auction L with $l_1 > l_1^{crit}$ but it must

be if $l_1 < l_1^{crit}$, the firm's decision rule to participate in auction L is given by

$$\varepsilon_1(l_1, m_1) = \begin{cases} \text{Enter Auction } L & \text{if } l_1 \leq g_1(m_1) \\ \text{Skip Auction } L & \text{if } l_1 > g_1(m_1) \end{cases}$$

Next we derive the equilibrium bidding functions for both procurement auctions since these determine expected profits on which the entry indifference curve $g_1(m_1)$ depends. Since the equilibrium bidding functions depend on the equilibrium entry indifference curve themselves, we derive these for any entry indifference curve $g_1(m_1)$. The equilibrium indifference curve is denoted by $g(m)$ and is identified given equilibrium bidding behavior. Thus equilibrium bidding and the entry indifference curve are simultaneously determined.

2.1 Equilibrium Bidding Functions

Both auctions employ the first-price sealed-bid auction format where the lowest bid wins.⁵ Under this auction design, the symmetric equilibrium strategy in a one-shot auction is well-known and summarized for the procurement context in lemma 1.

Lemma 1 *Cohen and Loeb (1990), Equilibrium bidding in a first-price sealed-bid procurement auction*

Let project completion costs of n risk-neutral firms that bid for a single project contract be private information and independently and identically distributed according to cdf $H(c)$, $c \in [\underline{c}, \bar{c}]$.

Then, the symmetric equilibrium bidding function is $b(c) = c + \int_c^{\bar{c}} [1 - H(x)]^{n-1} dx / [1 - H(c)]^{n-1}$. (for a proof see the referees' appendix)

For our sequential procurement auction game, we derive the equilibrium bidding function for each of the two project auctions by application of this lemma to our specific context with additional strategic interaction: In the first auction stage every firm knows that a second auction follows. In the second auction stage, each bidder receives information on the outcome of the first auction L .

Consider first the auction for project L . Any of the two firms that enters the first auction anticipates that in case it does not win the first auction, it will be the only bidder in the subsequent auction M where it will receive $\bar{c} - m$. Thus, it might submit a relatively high bid for project L , since it is, at least partially, insured against losing the first auction. In particular, a firm with a cost advantage for project M knows that the largest payoff it can receive is $\bar{c} - m$ from being the only bidder for project M since its largest payoff in auction L is $\bar{c} - l$ which must be smaller due to the firm's cost advantage. Thus, provided the firm decides to participate in

⁵Cf. Vickrey (1961), McAfee and McMillan (1987a) or Milgrom (1989) for a description of the first-price sealed-bid auction design.

auction L , it seeks to lose the first auction and minimizes its chances of winning project L by submitting the highest feasible bid which simultaneously maximizes its payoff from accidentally winning it. In contrast, if a firm has a cost advantage for project L , then it tops its completion cost l with its certain return from auction M and uses this revised cost parameter $\lambda \equiv l + \bar{c} - m$ in auction L . Intuitively it uses its total cost of executing project L that include the direct project cost l and the opportunity cost of winning project L , $\bar{c} - m$ (=benefit of not-winning auction L): Any firm taking part in auction L treats $\bar{c} - m$ as a safe profit. In case it wins project L , it pays the cost of executing this project and 'repays' the amount $\bar{c} - m$.

If there is no bidding competition in the auction for project M , then any firm bidding for it submits the maximum feasible bid to maximize profits. If it is not the only bidder then it receives the additional information that its competitor did not enter auction L , too. In response it updates its belief about its competitor's cost parameter for project M since skipping auction L might not be equilibrium behavior for every type. The appropriate a posteriori pdf is denoted by $f_{M|Skip}(m)$ and gives the (equilibrium) density that a firm with completion cost realization m for project M bids only in auction M . Put differently, $f_{M|Skip}(m)$ is the marginal pdf of $f(l, m)$ conditional on the fact that completion cost pair (l, m) leads the firm to skip auction L .

Proposition 2 *Equilibrium bidding functions in auctions L and M*

The equilibrium bidding functions of a firm with completion cost pair $(l, m) \in [\underline{c}, \bar{c}]^2$ are given by:

$$(a) \ b^L(\lambda) = \begin{cases} \bar{c} & \text{if } l \geq m \\ \lambda + \int_{\lambda}^{\bar{c}} [1 - F_{\lambda}(x)] dx / [1 - F_{\lambda}(\lambda)] & \text{otherwise} \end{cases}$$

if it submits a bid for project L where $\lambda \equiv l + \bar{c} - m$ and $F_{\lambda}(x) = \int_{\underline{c}}^x \int_{\bar{c} + \underline{c} - \lambda}^{\bar{c}} f(m - \bar{c} + \lambda, m) dm d\lambda$ with $x \in [\underline{c}, \bar{c}]$.

$$(b) \ b^M(m) = \begin{cases} \bar{c} & \text{if it is the only bidder} \\ m + \int_m^{\bar{c}} [1 - F_{M|Skip}(x)] dx / [1 - F_{M|Skip}(m)] & \text{otherwise} \end{cases}$$

if it submits a bid for project M where $f_{M|Skip}(x) = \left[\int_{g(x)}^{\bar{c}} f(l, x) dl \right] / \left[\int_{\underline{c}}^{\bar{c}} \int_{g(x)}^{\bar{c}} f(l, x) dl dx \right]$ and $F_{M|Skip}(x) = \int_{\underline{c}}^x f_{M|Skip}(s) ds$ and $g(x)$ denotes the competitor's entry indifference curve.

Proof See the appendix.

2.2 Entry Decision

In this section, we derive the entry indifference curve (1) under the assumption of equilibrium bidding that is summarized in proposition 2. Obviously the expected profit from bidding in auction L and possibly in auction M , $E[\Pi_1^{L+M} | (l_1, m_1)]$, always exceeds the expected profit from skipping auction L , $E[\Pi_1^M | m_1]$ if firm 1 has a cost advantage for project L (i.e. $l_1 \leq m_1$): the expected profit from entering auction L and possibly M is at least as large as $\bar{c} - m_1$. To see this suppose that the firm would bid \bar{c} in auction L . Then it receives in expectation

$(\bar{c} - l_1) \cdot \Pr(\text{won } L) + (\bar{c} - m_1) \cdot \Pr(\text{lost } L) \geq \bar{c} - m_1$. For any bid in auction M , the expected profit from skipping L , $E[\Pi_1^M | m_1]$, must be lower than $\bar{c} - m_1$ since there is a positive probability of bidding competition in auction M . It follows that (1) can only hold if $l_1^{crit} > m_1$ and that without loss of generality the entry indifference curve is defined by:

$$E \left[\Pi_1^{L+M} | (l_1^{crit} > m_1, m_1) \right] = E \left[\Pi_1^M | m_1 \right]. \quad (2)$$

In order to explicitly state equation (2), consider first its left-hand side. A firm with a cost advantage for project M that enters auction L bids \bar{c} in auction L and if it loses \bar{c} in auction M . This strategy results in four events summarized in the next table where the probabilities depend on firm 1's belief that firm 2 acts in accordance with the entry indifference curve $g_2(M_2)$.

| Events if firm 1 bids \bar{c} in auction L | Payoff | Probability |
|--|-----------------|---|
| A it is the only bidder | $\bar{c} - l_1$ | $\Pr(L_2 > g_2(M_2))$ |
| B the competitor bids \bar{c} and firm 1 wins L | $\bar{c} - l_1$ | $\Pr(g_2(M_2) \geq L_2 \geq M_2) \cdot 0.5$ |
| C the competitor bids \bar{c} and firm 1 loses L | $\bar{c} - m_1$ | $\Pr(g_2(M_2) \geq L_2 \geq M_2) \cdot 0.5$ |
| D the competitor bids less than \bar{c} | $\bar{c} - m_1$ | $\Pr(L_2 < M_2)$ |

Hence the expected benefit to a firm with a cost advantage for project M from starting bidding for project L with $b^L = \bar{c}$ and then continuing bidding in auction M after losing auction L is given by

$$\begin{aligned}
E \left[\Pi_1^{L+M} | (l_1 > m_1, m_1) \right] &= \int_{\underline{\epsilon}}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 \cdot (\bar{c} - l_1) & (A) \\
&+ \int_{\underline{\epsilon}}^{\bar{c}} \int_{m_2}^{g_2(m_2)} \frac{f(l_2, m_2)}{2} dl_2 dm_2 \cdot [2\bar{c} - (l_1 + m_1)] & (B+C) \\
&+ \int_{\underline{\epsilon}}^{\bar{c}} \int_{\underline{\epsilon}}^{m_2} f(l_2, m_2) dl_2 dm_2 \cdot (\bar{c} - m_1) & (D)
\end{aligned}$$

Now, consider the right-hand side of (2). If firm 1 skips auction L there are three events depending on the entry behavior of its competitor. Again firm 1 assesses the probabilities of these events given its belief about the competitor's entry indifference curve $g(M_2)$.

| Events if firm 1 skips auction L | Payoff | Probability |
|------------------------------------|------------------|--|
| E no bidding competition | $\bar{c} - m_1$ | $\Pr(L_2 \leq g_2(M_2))$ |
| F firm 1 wins project M | $b^M(m_1) - m_1$ | $\Pr(M_2 > m_1 \wedge L_2 > g_2(M_2))$ |
| G the competitor wins project M | 0 | $\Pr(M_2 < m_1 \wedge L_2 > g_2(M_2))$ |

Thus the expected benefit to firm 1 with a cost advantage for project M from skipping bidding for project L can be written as:

$$E[\Pi_1^M | m_1] = (\bar{c} - m_1) \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{g_2(m_2)} f(l_2, m_2) dl_2 dm_2 \quad (\text{E})$$

$$+ \int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} [b^M(m_1) - m_1] \cdot f(l_2, m_2) dl_2 dm_2 \quad (\text{F+G})$$

A standard result in auction theory is that in first-price sealed-bid auctions bids are formed such that they equal the expected second-order statistic from the relevant type pool (conditional on the own type being the first-order statistic). Thus, $b^M(m_1)$ in the last term in $E[\Pi_1^M | m_1]$ can be substituted by m_2 . This is formally confirmed by lemma 3 implying:

$$E[\Pi_1^M | m_1] = (\bar{c} - m_1) \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{g_2(m_2)} f(l_2, m_2) dl_2 dm_2 \quad (3)$$

$$+ \int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} (m_2 - m_1) \cdot f(l_2, m_2) dl_2 dm_2$$

Clearly $E[\Pi_1^M | m_1]$ decreases in m_1 and is independent of l_1 .

Lemma 3 *Firm 1's expected profit from equilibrium bidding in auction M if there is bidding competition and $b_1^M = b^M(m_1)$ is equal to the expected profit from bidding the expected second-order statistic of completion costs given that firm 1's completion costs are the lowest, i.e.*

$$\int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} (b_1^M - m_1) \cdot f(l_2, m_2) dl_2 dm_2 = \int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} (m_2 - m_1) \cdot f(l_2, m_2) dl_2 dm_2.$$

Proof See the appendix.

Lemma 4 *Boundaries of the entry indifference curve $g(m)$*

Let $l_1^{crit} = g_1(m_1)$ be implicitly defined by $E[\Pi_1^{L+M} | (l_1^{crit}, m_1)] = E[\Pi_1^M | m_1]$. If $g_1(m_1)$ exists, then $m_1 < g_1(m_1) < \bar{c}$ for $m_1 \in [\underline{c}, \bar{c})$ and $g_1(\bar{c}) = \bar{c}$.

Proof See the appendix.

In order to determine the entry indifference curve $l_1^{crit} = g_1(m_1)$ it is sufficient to consider entry condition (1) for firms with a cost advantage for project M since, according to lemma 4, for all other types expected profits strictly exceed opportunity costs. Thus, substitution of

the definitions of expected profits from both entry strategies into (2) leads with some minor algebraic manipulations⁶ using the properties of $f(l, m)$ to the implicit definition of the entry indifference curve $l_1^{crit} = g(m_1)$:

$$\frac{\bar{c} - m_1}{2} + \frac{2\bar{c} - (l_1^{crit} + m_1)}{2} \left[\frac{1}{2} - \int_{\underline{c}}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 \right] + (\bar{c} - l_1^{crit}) \int_{\underline{c}}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 \quad (4)$$

$$- (\bar{c} - m_1) \int_{\underline{c}}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 - \int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} (m_2 - m_1) \cdot f(l_2, m_2) dl_2 dm_2 = 0.$$

The existence of the entry indifference curve in symmetric equilibrium is verified in proposition 5 where also its properties are given. Its proof contains a differential equation whose solution is the symmetric equilibrium indifference curve $g(m)$. Figure 1 illustrates these results for a representative firm with completion cost pair (l, m) .

Proposition 5 *For the symmetric perfect Bayesian equilibrium characterized by the representative firm's strategy $[b^L(l, m), b^M(m), \varepsilon(m)]$ and the density function $f(l, m)$:*

(a) *There exists a (nonempty) compact and convex set of completion cost pairs where it is rational for a firm to bid for project L although it has a cost advantage for completing project M. This subset is defined by $G = \{(l, m) \in [\underline{c}, \bar{c}]^2 \mid m \leq l \leq g(m)\}$.*

(b) The critical value function $g(m)$ exists and

(i) $m < g(m) < \bar{c}$ if $m \in [\underline{c}, \bar{c})$, $g(\bar{c}) = \bar{c}$, $g(\underline{c}) > \underline{c}$,

(ii) $g'(m) > 0$,

(iii) $g''(m) < 0$ f. $m \in [\underline{c}, \bar{c})$ and $g''(\bar{c}) = 0$.

Proof

Part (b) of the proposition directly implies the existence and the claimed properties of set G in (a). Therefore it is sufficient to show the existence of $g(m)$ satisfying (i)-(iii) to prove the proposition.

The implicit-function theorem implies existence and differentiability of $l_1^{crit} = g_1(m_1)$ as defined by (4) for any function $g_2(m_2)$. In a symmetric equilibrium, both firms act in accordance with the same equilibrium entry indifference curve denoted by $g(m)$. Thus $g_1(m) = g_2(m) = g(m)$ and (i) follows from lemma 4. Again, using symmetry of equilibrium entry behavior with identity

⁶See the referees' appendix.

(4) and subsequent differentiation w.r.t. m leads to

$$-\frac{g'(m)}{2} \int_{\underline{c}}^{\bar{c}} \int_{g(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 - \frac{g'(m)}{4} + \frac{1}{4} - \frac{1}{2} \int_{\underline{c}}^{\bar{c}} \int_{g(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 \quad (5)$$

$$+ \int_{\underline{c}}^{\bar{c}} \int_{g(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 = 0.$$

By (i) $\int_{\underline{c}}^{\bar{c}} \int_{g(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 < \int_{\underline{c}}^{\bar{c}} \int_{m_2}^{\bar{c}} f(l_2, m_2) dl_2 dm_2$ where the latter equals 1/2, identity (5) implies $g'(m) > 0$ as claimed in (ii).

Differentiation of (5) w.r.t. m yields

$$-\frac{g''(m)}{2} \int_{\underline{c}}^{\bar{c}} \int_{g(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 - \frac{g''(m)}{4} - \int_{g(m)}^{\bar{c}} f(l_2, m) dl_2 = 0$$

implying (iii). \square

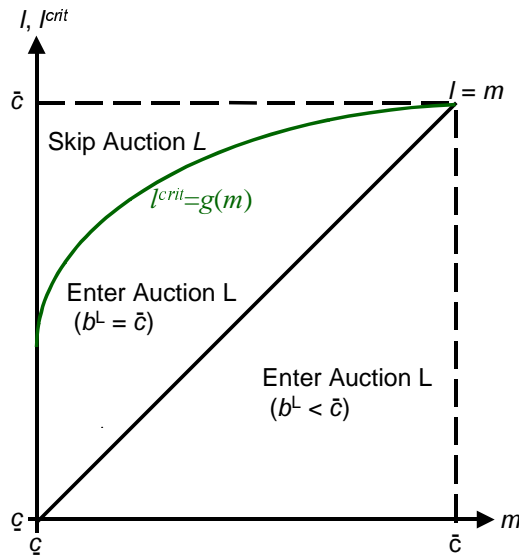


Figure 1: Equilibrium Entry Behavior

Consideration of proposition 5 leads to the conclusion that any firm always enters auction L if it faces a cost advantage for this project, i.e. $l \leq m$. Then it earns at least $\bar{c} - m$ while skipping auction L leaves it with running the risk of lower profits in case its competitor also skipped the first auction resulting in lower expected profits of this strategy. In contrast, a cost advantage for completing project M implies the impossibility of the firm to secure itself the same return in auction L as it could earn in auction M being the only bidder. However, if the competing firm has a strong cost advantage for project M and skipped auction L , then there is competition in

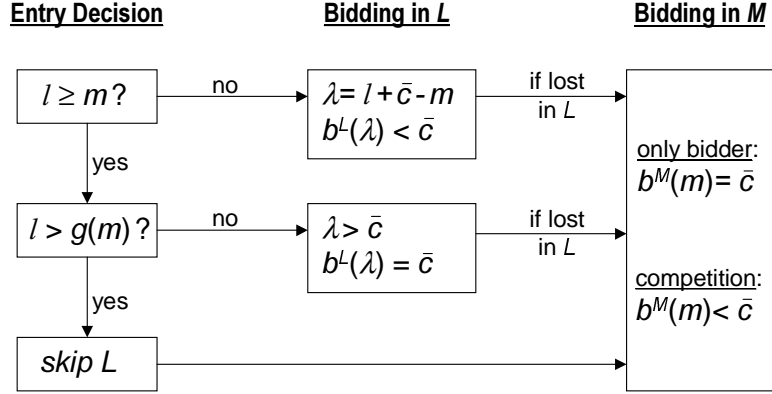


Figure 2: Equilibrium structure of the sequential procurement auction game

the second auction with the risk of low or even zero profits due to aggressive bidding. Therefore a firm may wish to participate in the first auction and win the unloved project L at a high price to insure itself against low profits resulting from fierce competition in the second auction, although it actually prefers losing the auction for project L . Figure 2 illustrates the equilibrium structure of the game.

3 Revenue Equivalence

In this section we briefly demonstrate that our results on equilibrium entry with a first-price sealed-bid auction design coincide with those obtained under a second-price sealed-bid auction regime.⁷ Here the term revenue equivalence refers to the bidders' perspective and means that under revenue-equivalent auction formats, which may even differ across both auction stages, a bidding firm receives the same expected revenue net of expected project completion costs from equilibrium bidding.

Lemma 6 *Second-price sealed-bid auction: equilibrium bidding functions*

The equilibrium bidding functions of a firm with completion cost pair $(l, m) \in [\underline{c}, \bar{c}]^2$ under a second-price sealed-bid design are given by:

$$(a) \ b^L(l, m) = \begin{cases} \bar{c} - m + l & \text{if } l \leq m \\ \bar{c} & \text{if } l > m \end{cases} \quad \text{if it submits a bid for project } L.$$

$$(b) \ b^M(m) = m \quad \text{if it submits a bid for project } M.$$

Proof See the appendix.

Expected profits under the second-price sealed-bid design for the entry-strategies "skip auction L " and "enter auction L and possibly M " can be determined with the bidding functions as

⁷Cf. Gale and Hausch (1994) on the second-price sealed-bid design in a similar setting.

given in lemma 6 analogously to the profit determination with the first-price sealed-bid auction format:

$$E[\Pi_1^{M,SP} | m_1] = (\bar{c} - m_1) \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{g_2(m_2)} f(l_2, m_2) dl_2 dm_2 \quad (6)$$

$$+ \int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} (m_2 - m_1) \cdot f(l_2, m_2) dl_2 dm_2$$

$$E[\Pi_1^{L+M,SP} | (l_1 > m_1, m_1)] = (\bar{c} - m_1) \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{m_2} f(l_2, m_2) dl_2 dm_2 \quad (7)$$

$$+ \frac{2\bar{c} - (l_1 + m_1)}{2} \int_{\underline{c}}^{\bar{c}} \int_{m_2}^{g_2(m_2)} f(l_2, m_2) dl_2 dm_2 + (\bar{c} - l_1) \int_{\underline{c}}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2$$

Lemma 7 *Firm 1's expected profits from both mutually exclusive entry strategies, either entering auction L and possibly auction M or only bidding for auction M, coincide under the first-price and second-price sealed-bid auction formats if the entry indifference curves of firm 1's competitor coincide under both auction designs, i.e. for $g_2^{fp}(x) = g_2^{sp}(x) = g_2(x)$ it follows:*

- (a) $E[\Pi_1^{M,fp} | m_1] = E[\Pi_1^{M,sp} | m_1]$ where the RHS is defined by (6),
- (b) $E[\Pi_1^{L+M,sp} | (l_1 > m_1, m_1)] = E[\Pi_1^{L+M,sp} | (l_1 > m_1, m_1)]$ where the RHS is defined by (7).

Proof See the appendix.

Proposition 8 *The equilibrium entry indifference curve $g^{sp}(m)$ under the second-price sealed-bid auction format in the symmetric perfect Bayesian equilibrium is identical to the entry indifference curve $g^{fp}(m)$ under the first-price sealed-bid auction design as identified and characterized in proposition 5. Therefore, auction entry decisions coincide in equilibrium under both designs and there is equivalence of firms' expected equilibrium profits.*

Proof As for the first-price sealed-bid auction design, the entry indifference curve under the second-price auction format is defined by the equality of $E[\Pi_1^{M,sp} | m_1]$ and $E[\Pi_1^{L+M,sp} | (l_1 > m_1, m_1)]$ arising from participation in auction L and possibly in auction M. ($E[\Pi_1^{L+M,sp} | (l_1 \leq m_1, m_1)]$ can be neglected for exactly the same reasons as in the first-price sealed-bid case since lemma 4 applies here, too.) It follows that the only difference between the implicit definitions of the entry indifference curve under both auction formats is that under the second-price sealed-bid auction it is named $g_2^{sp}(m)$ as a result from $E[\Pi_1^{M,sp} | m_1]$ and $E[\Pi_1^{L+M,sp} | (l_1 > m_1, m_1)]$ as given in lemma 7. Therefore the entry indifference curves must be the same, i.e. $g_2^{sp}(m) = g_2^{fp}(m) = g_2(m)$. In consequence, proposition 5 also applies to the second-price sealed-bid auction format since no other property specific to the first-price sealed bid auction in its proof is used. However, the appropriate equilibrium bidding functions are given in lemma 6. \square

4 A Parameterized Example

In order to illustrate the results of our model, we utilize a bivariate uniform distribution to explicitly determine equilibrium entry and bidding strategies under the first-price sealed-bid auction design with:

$$f(l_2, m_2) = \begin{cases} \frac{1}{(\bar{c} - \underline{c})^2} & \text{if } \underline{c} \leq l_2 \leq \bar{c}, \underline{c} \leq m_2 \leq \bar{c} \\ 0 & \text{otherwise} \end{cases}.$$

Since bidding behavior depends on the entry indifference curve, we begin with the derivation of $g(m)$. Equation (5) implicitly defines $g(m)$ and simplifies with our distributional assumption to the following differential equation:

$$-\frac{g'(m)}{2\delta} \int_{\underline{c}}^{\bar{c}} [\bar{c} - g(m_2)] dm_2 - \frac{g'(m)}{4} + \frac{1}{4} - \frac{1}{2\delta} \int_{\underline{c}}^{\bar{c}} [\bar{c} - g(m_2)] dm_2 + \frac{1}{\delta} \int_m^{\bar{c}} [\bar{c} - g(m_2)] dm_2 = 0$$

with $\delta = (\bar{c} - \underline{c})^2$. This equation can be reduced to the following second-order differential equation

$$z''(m) - \frac{z(m)}{\delta v_1} - \frac{v_2}{v_1} = 0 \quad (8)$$

where

$$z(m) = \int_m^{\bar{c}} [\bar{c} - g(m_2)] dm_2, \quad (9)$$

$$v_1 = \left[\frac{z(\underline{c})}{2\delta} + \frac{1}{4} \right], \quad v_2 = \left[\frac{1}{4} - \frac{z(\underline{c})}{2\delta} \right]. \quad (10)$$

The general solution of (8) is

$$z(m) = A_1 e^{\frac{m}{\sqrt{\delta v_1}}} + A_2 e^{\frac{-m}{\sqrt{\delta v_1}}} - \delta v_2,$$

where A_1 and A_2 are arbitrary constants of integration. The fact that $g(\bar{c}) = \bar{c}$ implies $z'(\bar{c}) = 0$ and $z(\bar{c}) = 0$. In consequence, A_1 and A_2 are determined by the following system of equations:

$$\begin{aligned} z(\bar{c}) &= A_1 e^{\frac{\bar{c}}{\sqrt{\delta v_1}}} + A_2 e^{\frac{-\bar{c}}{\sqrt{\delta v_1}}} - \delta v_2 = 0 \\ z'(\bar{c}) &= \frac{A_1 e^{\frac{\bar{c}}{\sqrt{\delta v_1}}} - A_2 e^{\frac{-\bar{c}}{\sqrt{\delta v_1}}}}{\sqrt{\delta v_1}} = 0, \end{aligned}$$

with solution

$$A_1 = \frac{1}{2} \frac{\delta v_2}{e^{\bar{c}/\sqrt{\delta v_1}}}; \quad A_2 = \frac{1}{2} \delta v_2 e^{\bar{c}/\sqrt{\delta v_1}}.$$

Therefore, the definite solution of (8) is:

$$z(m) = \frac{1}{2} \delta v_2 \left[e^{\frac{m - \bar{c}}{\sqrt{\delta v_1}}} + e^{\frac{\bar{c} - m}{\sqrt{\delta v_1}}} - 2 \right].$$

Note that v_1 and v_2 depend on $z(\underline{c})$. For a given parameter \underline{c} , $z(\underline{c})$ can be numerically derived by solving the following equation w.r.t. $z(\underline{c})$.

$$z(\underline{c}) = \frac{1}{2}\delta v_2(z(\underline{c})) \left[e^{\frac{m-\bar{c}}{\sqrt{\delta v_1(z(\underline{c}))}}} + e^{\frac{\bar{c}-m}{\sqrt{\delta v_1(z(\underline{c}))}}} - 2 \right]. \quad (11)$$

By construction of (9), $g(m) = \bar{c} + z'(m)$ such that the entry indifference curve is given by

$$g(m) = \bar{c} + \frac{1}{2}\delta v_2 \left[\frac{e^{\frac{m-\bar{c}}{\sqrt{\delta v_1}}} - e^{\frac{\bar{c}-m}{\sqrt{\delta v_1}}}}{\sqrt{\delta v_1}} \right].$$

For the remainder of this example, let the domain of project costs be given by $[\underline{c}, \bar{c}]^2 = [20, 100]^2$ implying $\delta = 6400$, $z(20) = 1475.4373$, $v_1 = 0.365$ and $v_2 = 0.135$. For this distribution, the entry indifference curve is given by

$$g(m) = 100 + 8.938 \cdot \left[e^{\frac{m-100}{48.332}} - e^{\frac{100-m}{48.332}} \right].$$

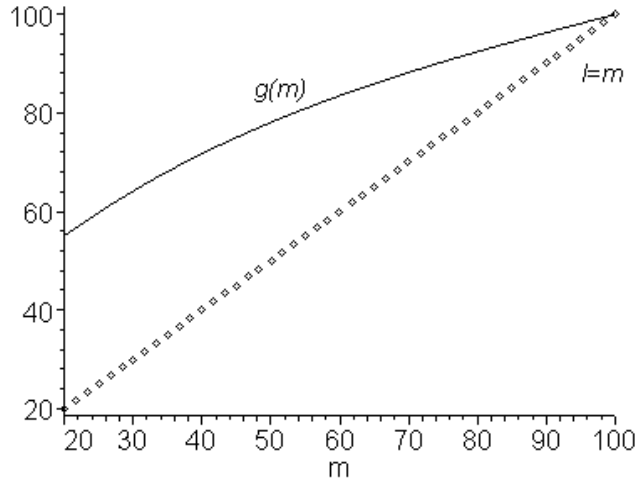


Figure 3: $g(m)$ for $(l, m) \sim U[20, 100]^2$

The cdf of opportunity-cost-augmented completion cost levels of project L is given by

$$F_\lambda(x) = \frac{(\lambda - 20)^2}{12,800}.$$

Using this result and the numerical function $g(m)$ with proposition 2a leads to the equilibrium bidding strategy in auction L :

$$b^L(\lambda) = \begin{cases} 100 & \text{for } g(m) \geq l \geq m \\ \frac{2}{3} \frac{\lambda^3 - 30\lambda^2 - 1,660,000}{\lambda^2 - 40\lambda - 12,400} & \text{otherwise} \end{cases}.$$

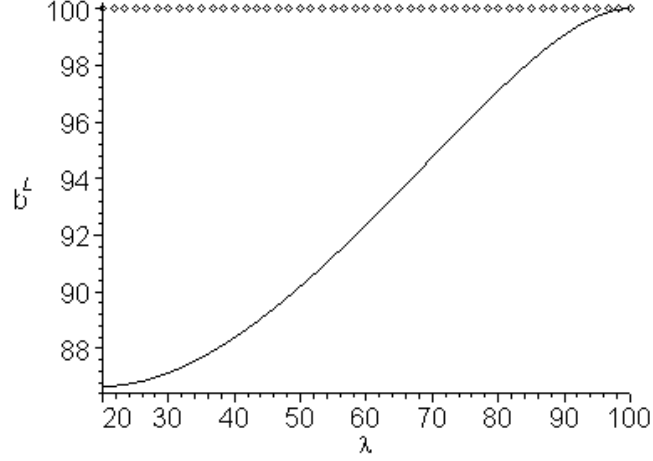


Figure 4: Equilibrium bidding function $b^L(\lambda)$

If both firms don't submit a bid in the first auction, each of them uses this information to update its beliefs about its competitor's distribution of completion costs resulting in the a posteriori density $f_{M|Skip}(x)$:

$$f_{M|Skip}(x) = 0.00604 \cdot \left[e^{\left[\frac{100-x}{48.332} \right]} - e^{\left[\frac{x-100}{48.332} \right]} \right].$$

According to proposition 2b, the bidding function of a firm that submits a bid in the second auction M is given by

$$b^M(m) = \begin{cases} 100 & \text{if it is the only bidder} \\ m + \frac{0.5839(m-100)+14.110 \left[e^{\left[\frac{100-m}{48.332} \right]} - e^{\left[\frac{m-100}{48.332} \right]} \right]}{0.292 \left[e^{\left[\frac{100-m}{48.332} \right]} + e^{\left[\frac{m-100}{48.332} \right]} \right] - 0.5839} & \text{otherwise} \end{cases}$$

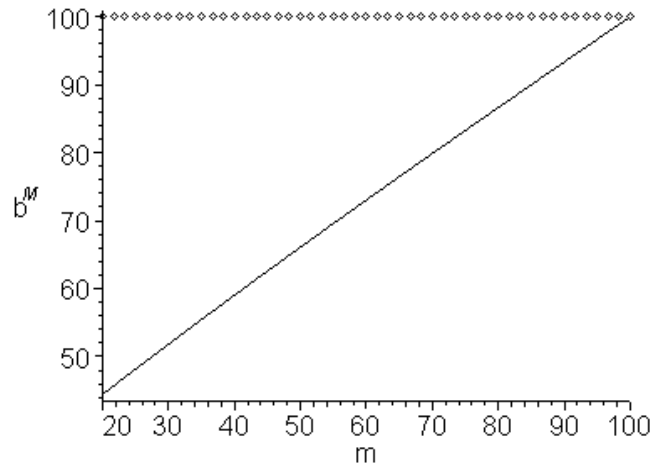


Figure 5: Equilibrium bidding function $b^M(m)$

5 Impact of Alternative Transaction Opportunities

In this section, we demonstrate how bidding behavior in the first auction varies with the value that a firm places on the opportunity to participate in a second auction. The option value negatively depends on a firm's completion cost for the second project and from the equilibrium bidding function $b^L(\lambda)$ with $\lambda = l + \bar{c} - m$ we obtain

$$\frac{\partial b^L(l, m)}{\partial m} = \begin{cases} -\frac{f_\lambda(\lambda) \int_\lambda^{\bar{c}} [1 - F_\lambda(x)] dx}{[1 - F(\lambda)]^2} < 0 & \text{if } l < m \\ 0 & \text{otherwise} \end{cases}.$$

As the completion cost m increases, the option value of participating in the second auction, $\bar{c} - m$, decreases and firms with a cost advantage for the first project bid more aggressively in the auction for project L in contrast to the prediction of the standard SIPV model. For the purpose of illustration, consider our parameterized example with two firms where completion costs are uniformly distributed on $[l, m]$.

- (a) (Base Scenario): Suppose the completion cost for project M to be fixed at some level below maximum completion cost, say $m = 80$. The bidding function for the first project L is:

$$b^L(l, m = 80) = \begin{cases} \frac{2}{3} \cdot \frac{l^3 + 30l^2 - 1,664,000}{l^2 - 12,800} & l < 80 \\ 100 & l \geq 80 \end{cases}$$

Figure 6 illustrates that bids in the dynamic auction environment with $m = 80$ substantially exceed bids in a one-shot auction given by $b(l)$ below.

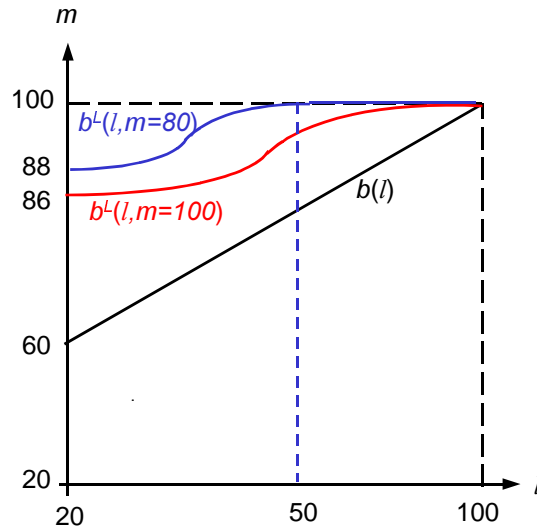


Figure 6: Bidding for project L under various outside options

- (b) (One-Shot-Auction) There is no second auction such that the first auction reduces to the standard SIPV model in a procurement context, see lemma 1:

$$b(l) = 50 + 0.5l$$

- (c) (Virtually No 2nd Auction) Consider base scenario (a) with maximum completion cost $m = 100$. Compared to $b^L(l, m = 80)$, the larger completion cost m shifts the bidding function $b^L(l, m = 100)$ downwards,

$$b^L(l, m = 100) = \frac{2}{3} \cdot \frac{l^3 - 30l^2 - 1,660,000}{l^2 - 12,400 - 40l}.$$

The exemplified change of $b^L(l, m)$ in response to increases in m illustrates typical comparative static behavior. Note that bids with virtually no second auction do not coincide with bids in the one-shot auction since completion costs are private information. It follows that the standard SIPV model is no special case of our dynamic auction model with an option value equal to zero. In addition, bidding for the second project under competition in our model is more aggressively than in the SIPV model for identical cost types m due to endogenous type selection: since relatively more low-cost types skip the first auction, in the second auction firm 1 expects its competitor to have a lower cost than it does in the SIPV model.

6 Conclusion

Motivated by recent empirical evidence that suggests that (1) firms perceive independent one-shot procurement auctions not in isolation but rather as an auction sequence, that (2) subcontracting isn't perfect and that (3) firms are aware of their opportunity cost, this paper has introduced a sequential procurement auction model to study firms' entry and bidding strategies. In contrast to a procurement auction version of Weber (1983) and Elmaghraby (2003), the presented model predicts that firms not always participate in early auctions. In addition bidding behavior in each auction stage strongly depends on the option value that a firm places on remaining auctions contrasting with a procurement auction version of the standard SIPV model. If real-world firms would regard auction stages in a sequence of auction as one-shot games and, thus, would ignore alternative transaction opportunities, the focus on the standard SIPV model is appropriate. The experimental data generated by the implementation of our procurement auction model in the laboratory, see Brosig and Reiß (2003), suggests that the option value of alternative transaction opportunities strongly influences bidding and entry behavior in the lab similarly to the theoretical predictions and renders our approach relevant.

7 Appendix

7.1 Proof of proposition 2

Consider first part (b): Before bidding for auction M , any firm knows if its competitor won auction L . If the competitor entered auction L , the firm remains the only bidder in auction M and maximizes its return by submitting the largest feasible bid equaling \bar{c} . If, however, its competitor skipped auction L , the firm infers that its competitor's completion cost pair must satisfy $L > g(M)$ (to be determined later) and Bayesian updating of the firm's cost belief regarding its competitor leads to the a posteriori pdf $f_{M|Skip}(x)$. Appealing to lemma 1 leads to $b^M(m)$.

For (a), note that a firm receives $\bar{c} - m$ in auction M if it loses auction L . If its completion costs satisfy $m \leq l$ then it cannot receive a larger return in auction L , $\bar{c} - m \geq \bar{c} - l$ by assumption. Thus the firm chooses to receive the largest feasible return from auction L by bidding \bar{c} which also maximizes the frequency it ends up with the larger return from auction M , provided it submits a bid for project L .

In case the firm has a cost advantage for project L , i.e. $l < m$, its expected profit from participating in auction L with any bid $b_1^L \in [c, \bar{c}]$ and possibly in auction M is given by

$$E[\Pi_1^{L+M} | l_1 < m_1] = (b_1^L - l_1) \cdot \Pr(b_1^L \text{ wins auction } L) + (\bar{c} - m_1) \cdot [1 - \Pr(b_1^L \text{ wins auction } L)].$$

Using firm 1's total cost parameter $\lambda_1 \equiv l_1 + \bar{c} - m_1$, this can be rewritten as

$$E[\Pi_1^{L+M} | l_1 < m_1] = (b_1^L - \lambda_1) \cdot \Pr(b_1^L \text{ wins auction } L) + \bar{c} - m_1$$

From firm 1's perspective $\bar{c} - m_1$ is a known constant and its expected profit from bidding in auction L , $E[\Pi_1^{L+M} | l_1 < m_1]$, is maximized if b_1^L maximizes

$$Z(\lambda_1) := (b_1^L - \lambda_1) \cdot \Pr(b_1^L \text{ wins auction } L) \tag{12}$$

where $l_1 < m_1 \Leftrightarrow \lambda_1 < \bar{c}$ by definition. Suppose there exists a symmetric equilibrium bidding function $b^L(\lambda)$ that maximizes Z such that it is strictly increasing for $\lambda < \bar{c}$, $b^L(\lambda) < \bar{c}$ for $\lambda < \bar{c}$, and $b^L(\lambda) = \bar{c}$ for $\lambda \geq \bar{c}$. Since $b^L(\lambda)$ is strictly increasing on $[c, \bar{c}]$, there exists an inverse on that domain denoted by $b^{-1,L}(b^L)$. Given that firm 1's competitor adheres to this equilibrium bidding function, firm 1 (with a cost advantage for project L) wins always the first round if its competitor has a cost advantage for project M , $\lambda_2 \geq \bar{c}$. It wins project L too, if it bids an amount that corresponds to a lower total cost type $\lambda_1 = b^{-1,L}(b^L)$ than the one of its competitor λ_2 . Denoting the cdf of total cost types by $F_\lambda(\lambda)$, (12) can be rewritten as⁸

$$Z(\lambda_1) = (b_1^L - \lambda_1) \cdot [1 - F_\lambda(b^{-1,L}(b_1^L))] \tag{13}$$

⁸Here the fact is used that types with a cost advantage for project L always enter the first auction. This is formally confirmed in lemma 4.

where $f_\lambda(\lambda) = \int_{\bar{c}+\underline{c}-\lambda}^{\bar{c}} f(m-\bar{c}+\lambda, m) dm$ and $F_\lambda(\bar{c}) = 1/2$. If b_1^{*L} maximizes Z then $\partial Z^*/\partial b_1^L = 0$ and differentiation of (13) at the optimum w.r.t. λ_1 yields

$$\frac{dZ^*}{d\lambda_1} = - [1 - F_\lambda(b^{-1,L}(b_1^{*L}))].$$

Integration in the boundaries $[\lambda_1, \bar{c}]$ together with the fact that in a Nash equilibrium b_1^{*L} must coincide with the value of the equilibrium bidding function at the true cost type λ_1 leads to

$$Z^*(\bar{c}) - Z^*(\lambda_1) = - \int_{\lambda_1}^{\bar{c}} [1 - F_\lambda(x)] dx.$$

Since $b^L(\bar{c}) = \bar{c}$, we have $Z^*(\bar{c}) = 0$ and obtain with (13) at its optimum the equilibrium bidding function for $\lambda_1 < \bar{c}$:

$$b^L(\lambda_1) = \lambda_1 + \frac{\int_{\lambda_1}^{\bar{c}} [1 - F_\lambda(x)] dx}{1 - F_\lambda(\lambda_1)}$$

□

7.2 Proof of lemma 3

Noting that m_1 and b_1^M are constants and substitution for the latter according to proposition 2 leads to

$$\frac{\int_{m_1}^{\bar{c}} [1 - F_{M_2|Skip}(x)] dx}{1 - F_{M_2|Skip}(m_1)} \cdot \int_{m_1}^{\bar{c}} \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 = \int_{m_1}^{\bar{c}} (m_2 - m_1) \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2$$

Since $1 - F_{M_2|Skip}(x) = \left[\int_x^{\bar{c}} \int_{g_2(s)}^{\bar{c}} f(l_2, s) dl_2 ds \right] / \left[\int_{\underline{c}}^{\bar{c}} \int_{g_2(t)}^{\bar{c}} f(l_2, t) dl_2 dt \right]$ we obtain from the last equation

$$\int_{m_1}^{\bar{c}} \int_x^{\bar{c}} \int_{g_2(s)}^{\bar{c}} f(l_2, s) dl_2 ds dx = \int_{m_1}^{\bar{c}} (m_2 - m_1) \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2.$$

Let $\phi(m_2)$ be the integral of $\int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2$, then integration by parts of the right-hand side using $\int \int_{g_2(m_2)}^{\bar{c}} f(l_2, m_2) dl_2 dm_2 = \phi(\bar{c}) - \int_{m_2}^{\bar{c}} \int_{g_2(u)}^{\bar{c}} f(l_2, u) dl_2 du$ leads to

$$\int_{m_1}^{\bar{c}} \int_x^{\bar{c}} \int_{g_2(s)}^{\bar{c}} f(l_2, s) dl_2 ds dx = \int_{m_1}^{\bar{c}} \int_{m_2}^{\bar{c}} \int_{g_2(s)}^{\bar{c}} f(l_2, s) dl_2 ds dm_2$$

which obviously holds. □

7.3 Proof of lemma 4

Suppose $g_1(m_1) = \bar{c}$, then firm 1 always enters auction L . For $l_1 = \bar{c}$, the firm never makes any profit from project L such that the strategy to enter auction L and continuing bidding in auction M if possible is profitable only if it doesn't win project L resulting in the same profit

as if the firm had skipped auction L . If the competitor has a cost advantage for project M , it either bids \bar{c} for project L or skips the auction. In both cases, firm 1 wins project L with positive probability although it could have won the profitable auction M in some cases since $m_1 < \bar{c}$ such that $E[\Pi_1^{L+M} | (l_1 = \bar{c}, m_1 < \bar{c})] < E[\Pi_1^M | m_1 < \bar{c}]$. Since $g_1(m_1) > \bar{c}$ leads to the same entry strategy as $g_1(m_1) = \bar{c}$ implies, it follows that $g_1(m_1) < \bar{c}$ for $m_1 \in [\underline{c}, \bar{c})$.

For $g_1(m_1) < \bar{c}$ it follows immediately from (3) that $E[\Pi_1^M | m_1 < \bar{c}] < \bar{c} - m_1$. If firm 1 has no cost advantage for project M , i.e. $l_1 \leq m_1$, according to proposition 2 its bidding behavior guarantees it at least profit $\bar{c} - m_1$ for each feasible completion cost pair of its competitor, thus $E[\Pi_1^{L+M} | l_1 \leq m_1 < \bar{c}] \geq \bar{c} - m_1$. Therefore, $g_1(m_1) > m_1$, $m_1 \in [\underline{c}, \bar{c})$.

If $m_1 = \bar{c}$, then the definition of $g_1(m_1)$ directly implies $g_1(\bar{c}) = \bar{c}$. To see this note that $E[\Pi_1^M | m_1 = \bar{c}] = 0$ and $E[\Pi_1^{L+M} | (\bar{c}, \bar{c})] = 0$ while $E[\Pi_1^{L+M} | (l_1 < \bar{c}, \bar{c})] > 0$ which completes the proof. \square

7.4 Proof of lemma 6

To prove (b) consider a Vickrey argument: b^M only deviates in payoff for the firm if overbidding with \hat{b} leads a competitor to win with bid \bar{b} where $\hat{b} > \bar{b} > m$ or underbidding with \hat{b} to win project M where $\hat{b} < \bar{b} < m$. In the first case, $b^M = m$ leads to the additional payoff $\bar{b} - m$, in the second case, underbidding results in the loss $m - \bar{b}$ while $b^M = m$ guarantees zero profits. Since in all other cases the strategies over- and underbidding and the bidding strategy $b^M = m$ lead to identical payoffs, the latter is a weakly dominant strategy in auction stage M .

For (a), at first consider $l \leq m$: If both firms submit bids for project L , the representative firm anticipates that it receives $\bar{c} - m$ in the second auction if it doesn't win project L . Thus, true completion costs including opportunity costs are given by $\lambda = \bar{c} - m + l$. A Vickrey argument analogous to the one given for (b) reveals optimality of $b^L = \lambda$. If the representative firm is the only bidder for project L , any bid guarantees it the surplus $\bar{c} - l$, therefore bidding λ is a weakly dominant strategy if $l \leq m$. Now suppose $l > m$: Clearly, the representative firm cannot secure itself return $\bar{c} - m$ in auction L since this requires a bid exceeding \bar{c} . If, however, the firm submits a bid for project L , it submits the largest feasible bid, i.e. \bar{c} . Depending on the bidding behavior of the other firm, it receives either $\bar{c} - l$ or $\bar{c} - m$ where the latter exceeds the former. If it submits any lower bid in auction L than \bar{c} , it reduces its expected profit since it receives a lower return from bidding in auction L and increases the probability of winning the low prize. Thus bidding $b^L = \bar{c}$ for $l > m$ is optimal. \square

7.5 Proof of lemma 7

Proving (a) amounts to show that expected profits of firm 1 under both auction designs coincide for any competitor's entry indifference curve $g_2(m_2)$ if firm 1 adopts equilibrium bidding. In

either auction format, firm 1 receives the amount $\bar{c} - m_1$ if it happens to be the only bidder and nothing if its competitor has lower completion costs for project M (see lemma 6 and proposition 2b). Therefore it is sufficient to show that expected profits in both auction formats are the same conditional on bidding competition. Thus we have to demonstrate that the following equation holds b_1^M is firm 1's equilibrium bid under the first-price sealed-bid auction:

$$\int_{m_1}^{\bar{c}} \int_{g_2^{fp}(m_2)}^{\bar{c}} (b_1^M - m_1) \cdot f(l_2, m_2) dl_2 dm_2 = \int_{m_1}^{\bar{c}} \int_{g_2^{sp}(m_2)}^{\bar{c}} (m_2 - m_1) \cdot f(l_2, m_2) dl_2 dm_2$$

Since $g_2^{fp}(m_2) = g_2^{sp}(m_2)$, we can appeal to lemma 3 to prove that the preceding equality holds and thus part (a) of this lemma is established.

Now consider (b): According to proposition 2 and lemma 6, firm 1 bids in auction L under the first-price and the second-price auction design always maximum completion costs \bar{c} (since $l_1 > m_1$). It wins this auction and receives always $\bar{c} - l_1$ if its competitor skipped the auction, i.e. $L_2 > g_2^{fp}(M_2)$ or it receives this amount half the time and otherwise $\bar{c} - m_1$ if firm 2 has a cost advantage for project M and bids \bar{c} , occurring if $g_2^{fp}(M_2) \geq L_2 \geq M_2$. If firm 2 has a cost advantage for project L , it submits a bid below \bar{c} and firm 1 loses this auction but wins in turn project M giving it $\bar{c} - m_1$. Inspection of $E[\Pi_1^{L+M,sp} | (l_1 > m_1, m_1)]$ in (7) and $E[\Pi_1^{L+M,fp} | (l_1 > m_1, m_1)]$ in subsection 2.2 shows that firm 1 faces the same return for every competitor's type regardless of whether a first-price or second-price auction is employed if entry indifference curves coincide. \square

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